Macroeconomics - Data & Theory

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How to isolate the business cycle component?

Easy way is to use the Hodrick-Prescott (HP) filter

$$\min_{\{x_{\tau,t}\}_{t=1}^{T}} \sum_{t=2}^{T-1} (x_t - x_{\tau,t})^2 + \lambda \sum_{t=2}^{T-1} \left\{ [(x_{\tau,t+1} - x_{\tau,t}) - (x_{\tau,t} - x_{\tau,t-1})]^2 \right\}$$

Filtered series is then defined as

$$x_{hp,t} = x_t - x_{\tau,t}$$

When $\lambda = 1,600$, then the filter turns out to be similar to a frequency domain filter that takes out all frequencies associated with cycles that have a period that exceeds 32 quarters.

How to isolate the business cycle component in Matlab?

- Suppose "data" is a $T \times n$ matrix with observations for n variables
- HP-*trend* is given by

  datatrend $=$ hpfilter(data,1600);

- Business cycle component is given by

  databc $=$ data-datatrend;

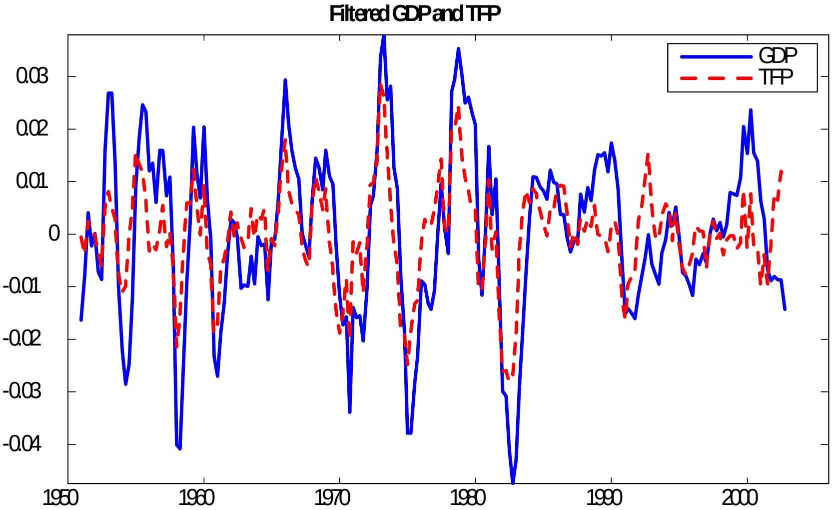
Key stylized facts (1951Q1-2005Q4)

x_t (all in real terms)	σ_x	Cor(x_{t-1},y_t)	Cor(x_t,y_t)	Cor(x_{t+1},y_t)
ln(GDP)=y	1.57%	0.84	1	0.84
ln(non-dur. cons.)	1.09%	0.74	0.78	0.68
ln(dur. cons.)	4.86%	0.71	0.75	0.59
ln(priv. invest.)	4.89%	0.77	0.86	0.78
ln(priv. n-r. invest.)	4.83%	0.54	0.77	0.84
ln(priv. r. invest.)	9.48%	0.72	0.61	0.38
ln(wage rate)	0.87%	0.25	0.31	0.29
ln(GDP/person)	1.31%	0.72	0.69	0.34
ln(GDP/hour)	1.04%	0.55	0.52	0.17
ln(TFP) - 1951Q1-2002Q2	0.93%	0.73	0.74	0.43
unemploy. rate	0.72%	-0.66	-0.86	-0.88
employment rate	0.58%	0.59	0.79	0.84
participation rate	0.24%	0.19	0.30	0.39
Δinventory/GDP	0.41%	0.47	0.59	0.38
investment/GDP	0.46%	0.69	0.73	0.70

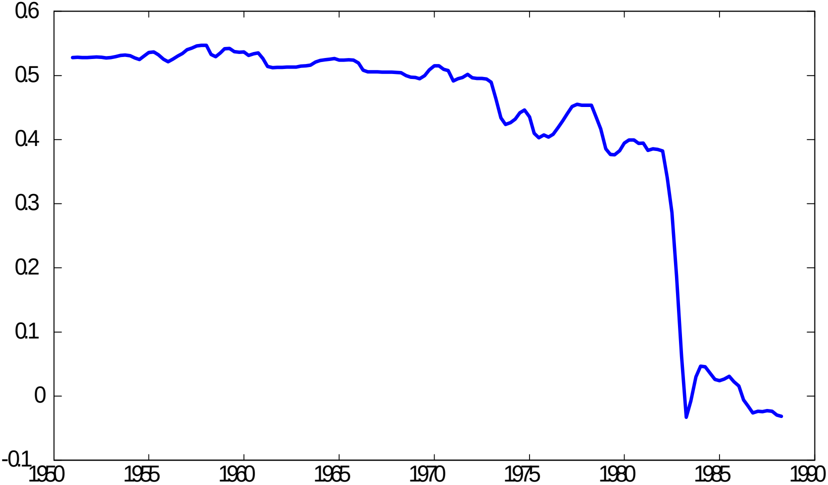
In words

- Output more volatile than TFP
- This measure of TFP not adjusted for capacity utilization, thus ...
- GDP/hour less volatile than GDP/person, why?
- Residential investment very volatile
- Unemployment rate = #unemployed/labor force
- Employment rate = #employed/population (>16yr) $\neq$ 1-unemp rate
- One small component is key for business cycle fluctuations
- Wages more cyclical if corrected for composition effects
- high autocorrelation coefficients

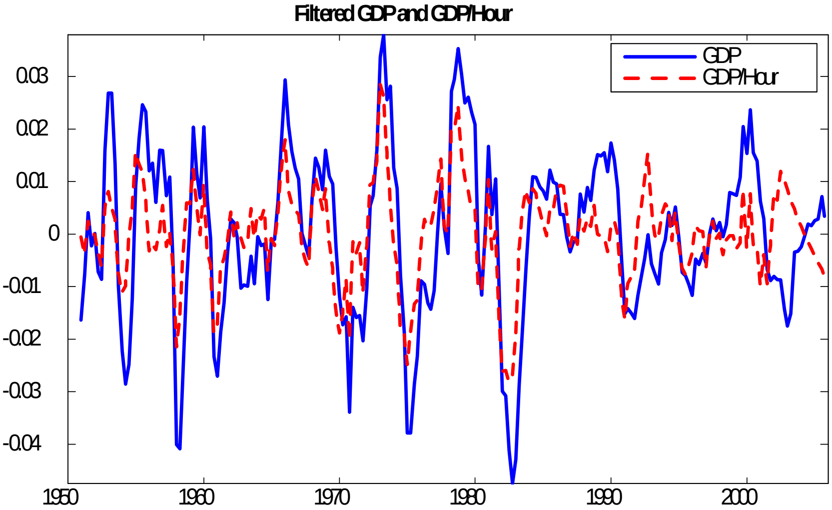
Correlation of GDP and TFP has declined



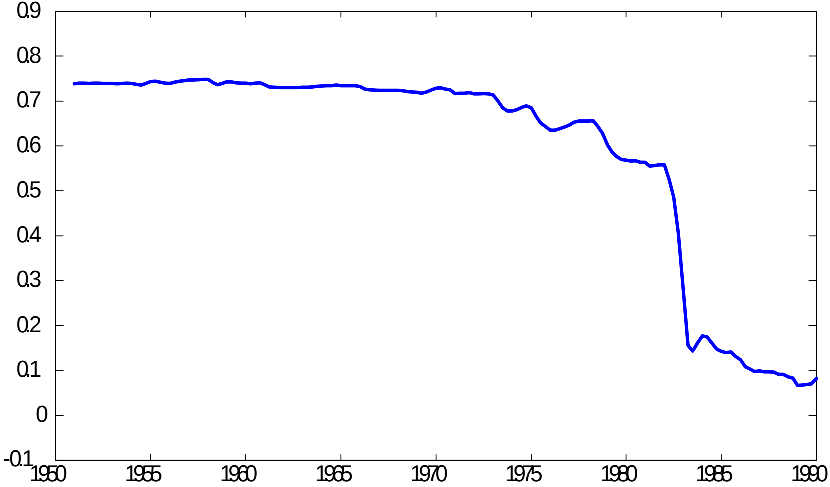
Correlation GDP & TFP from x value till '02Q2



Same for GDP & GDP/Hours



Same for GDP & GDP/Hours



Standard RBC model with leisure

$$\max_{\{c_{t+j},k_{t+1+j},h_{t+j}\}_{j=0}^{\infty}} \mathsf{E}\left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, 1-h_{t+j})|I_t\right]$$

$$\text{s.t. } c_{t+j} + k_{t+1+j} \leq w_{t+j}h_{t+j} + r_{t+j}^k k_{t+j} + (1-\delta)k_{t+j}$$

$$k_{t+1+j} \geq 0$$

$$k_t \text{ predetermined}$$

First-order conditions Households

$$\frac{\partial u(c_t, 1-h_t)}{\partial c_t} w_t = \frac{\partial u(c_t, l_t)}{\partial l_t} \qquad l_t = 1 - h_t$$

$$\frac{\partial u(c_t, 1-h_t)}{\partial c_t} = \mathsf{E}_t \left[\beta \frac{\partial u(c_{t+1}, 1-h_t)}{\partial c_{t+1}} (1 - \delta + r^k_{t+1} \right]$$

Firm problem & first-order conditions

$$\max_{\{h_t,k_t\}} \theta_t k_t^{\alpha} h_t^{1-\alpha} - w_t h_t - r_t^k k_t$$

$$w_t = (1-\alpha)\theta_t k_t^{\alpha} h_t^{-\alpha}$$

$$r_t^k = \alpha \theta_t k_t^{\alpha-1} h_t^{1-\alpha}$$

What if wages are acyclical?

$$\frac{\partial u(c_t, l_t)}{\partial c_t} w_t = \frac{\partial u(c_t, l_t)}{\partial l_t}$$

- Suppose $u_c \uparrow \Longrightarrow u_l \uparrow$
- Suppose $u_c \downarrow \Longrightarrow u_l \downarrow$
- Thus if consumption is procyclical then hours are countercyclical
- Thus if consumption is procyclical then hours are countercyclical

Special case with analytical solution

Log utility: $u(c_t, l_t) = \ln(c_t) + B\ln(l_t)$
Complete depreciation: $\delta = 1$
First-order conditions:

$$\frac{1}{c_t} = \mathsf{E}_t\left[\beta\frac{1}{c_{t+1}}\alpha\theta_{t+1}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha-1}\right]$$

$$\frac{w_t}{c_t} = \frac{B}{1-h_1}$$

Solution to first-order conditions

- Similar to model in notes without leisure

$$
\begin{aligned}
c_t &= (1-\alpha\beta)y_t = (1-\alpha\beta)k_t^{\alpha}h_t^{1-\alpha} \Longrightarrow \\
\frac{c_t}{y_t} &= (1-\alpha\beta)
\end{aligned}
$$

- Plug this into first-order condition for labor/leisure

$$
\begin{aligned}
\frac{(1-\alpha)y_t/h_t}{(1-\alpha\beta)y_t} &= \frac{B}{1-h_t} \Longrightarrow \\
h_t &= \bar{h} = \frac{1-\alpha}{B(1-\alpha\beta)+(1-\alpha)}
\end{aligned}
$$

- Thus the savings rate and hours decision are constant

Failures of the special case

- Savings rate and hours decision are constant
- Wage rate is too volatile and too procyclical

$$\ln w_t = \ln(1-\alpha) + \ln y_t - \ln \bar{h}$$

- Consumption is too volatile and too procyclical

$$\ln c_t = \ln(1-\alpha\beta) + \ln y_t$$

- Investment is not volatile enough

$$\ln i_t = \alpha\beta + \ln y_t$$

- Observations are related! More volatile investment requires procyclical savings rate

What about propagation?

- Data: TFP has less persistence than output
  - For example, $\rho(\Delta\theta_t, \Delta\theta_{t-1}) = 0.11$ and $\rho(\Delta y_t, \Delta y_{t-1}) = 0.32$ - (1951Q1-2002Q2)
- Does the model endogenously generate more persistence?
  - For capital?

$$\ln k_{t+1} = \text{constant} + \ln\theta_t + \alpha \ln k_t$$

  - For output?

$$\begin{aligned} \ln y_t &= \text{constant} + \ln\theta_t + \alpha \ln k_t \\ &= \text{constant} + \ln\theta_t + \alpha \ln y_{t-1} \end{aligned}$$

Propagation for capital survives for realistic depreciation, but result for output does not.

Shocks versus the model

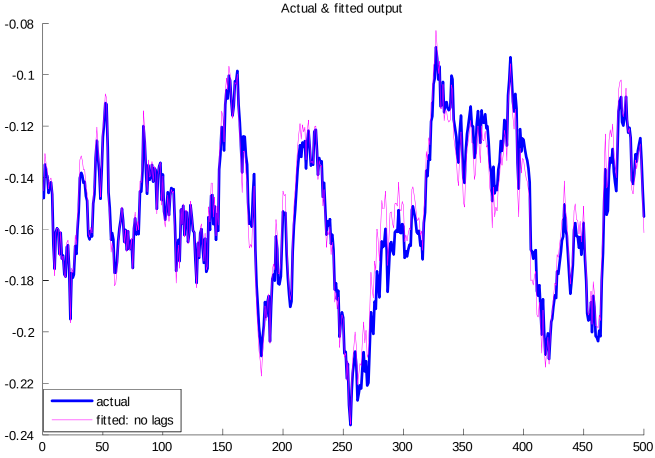
Policy rule in DSGE model:

$$z_{t+1} = a_0 + A_1 z_t + A_2 shocks_t$$

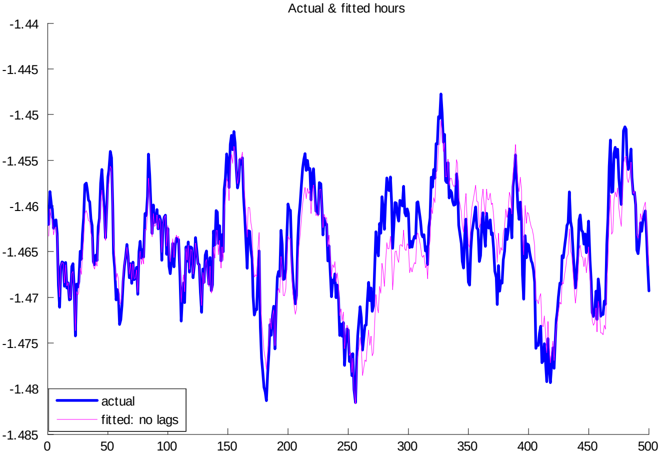
- How much can the contemporaneous values of the shocks explain by themselves?
- That is, how bad is this SGE model:

$$z_{t+1} = \tilde{a}_0 + \tilde{A}_2 shocks_t$$

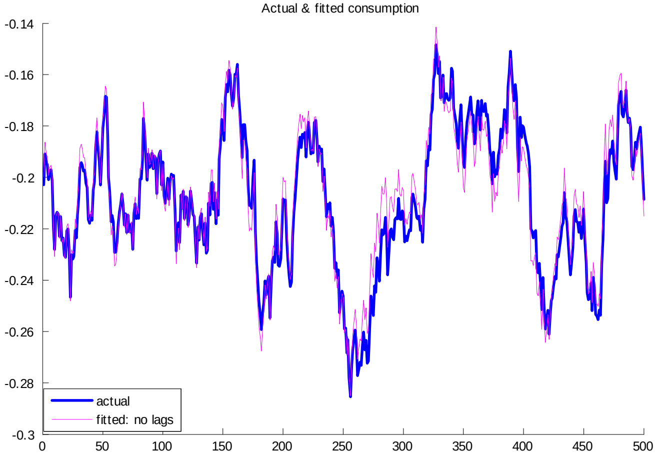
Output & current productivity shock



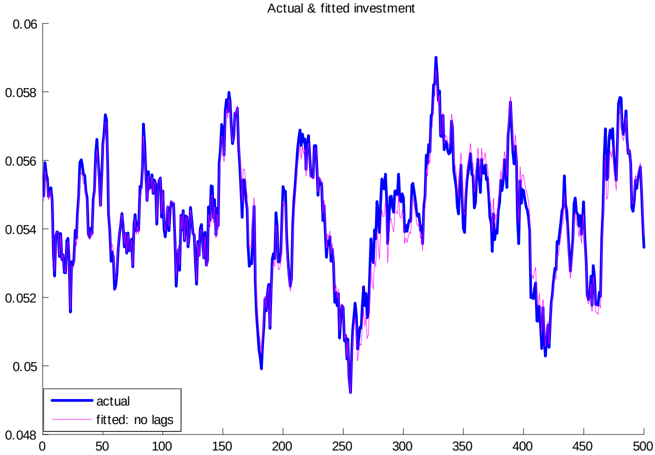
Hours & current productivity shock



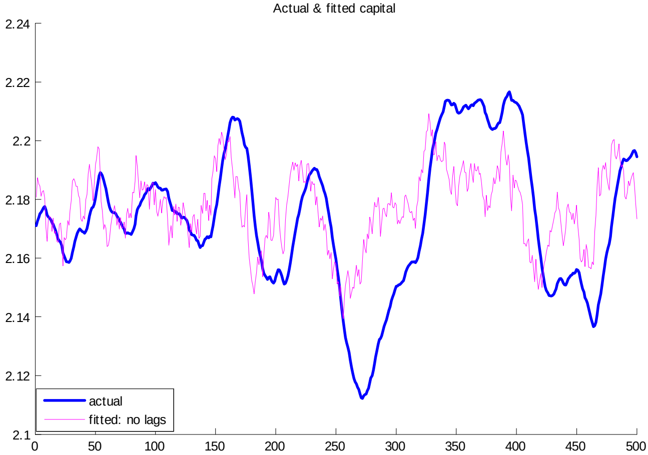
Consumption & current productivity shock



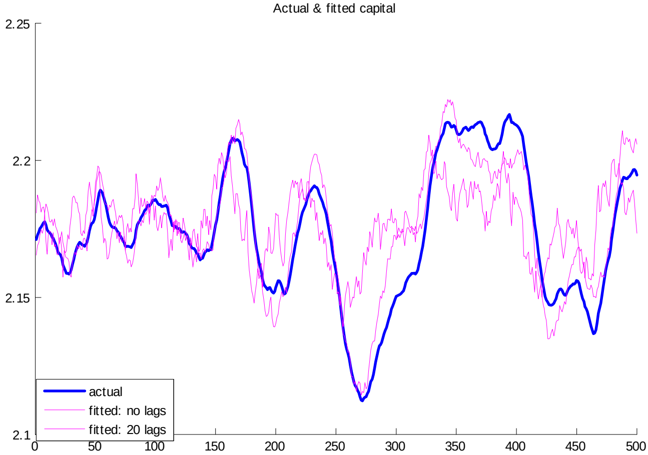
Investment & current productivity shock



Capital & current productivity shock



Adding 20 lagged values of the shock



Adding 40 lagged values of shock