Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty

Wouter J. DEN HAAN*, Kenneth L. JUDD†, and Michel JUILLARD‡

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Abstract

This paper describes the first model considered in the computational suite project that compares different numerical algorithms. It is an incomplete markets economy with a continuum of agents and an inequality (borrowing) constraint.

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*University of Amsterdam and CEPR; wdenhaan@uva.nl.

†Stanford University and NBER; judd@hoover.stanford.edu.

‡CEPREMAP and University Paris 8; wdenhaan@uva.nl.

1 Introduction

There has been an enormous amount of progress in the development of dynamic stochastic general equilibrium (DSGE) models. Whereas the first generation consisted of models with a representative agents and frictionless markets, recent models allow for much richer environments including, for example, heterogeneous agents, information asymmetries, contracting problems, matching frictions, incomplete markets, anticipated shocks, and adjustment costs. The complexity of the model makes it virtually impossible to analyze the properties of the model without relying on numerical solution techniques. The added complexity required the development of new algorithms. Moreover, several of the features introduced, especially idiosyncratic shocks and inequality constraints, increased the importance of nonlinearities, which obviously enhances the challenge to obtain an accurate numerical solution using a limited amount of computing time.

The model described in this paper is the first model used in the computational suite project that compares the properties of numerical algorithms to solve complex models. It is a popular model with a continuum of agents, idiosyncratic income shocks, incomplete financial markets, and aggregate uncertainty. The next model considered in the computational project focuses on models with a large finite number of agents.[1]

Neither model exhausts the list of features one would like to include in a model. Therefore, this is only the starting point. The study of the advantages and disadvantages of different algorithms should continue as models are getting more complex. We find it troublesome that the properties of numerical algorithms and especially the accuracy of numerical solution obtains so little attention by so many authors these days. It is, of course, not sensible to believe that the ability of an algorithm to obtain an accurate solution in a particular environment is a guarantee that the algorithm will generate an accurate solution in another environment and it is absurd to believe so when the environment is more complex. It is our sincere hope that this computational project will motivate economists to be more careful in their choice of the solution algorithm used and in particular in the

[1] In earlier versions of the model descriptions, the model described in this paper was referred to as "Model B" and the model with the large finite number of agents was referred to as "Model A".

evaluation of the accuracy of the numerical solution.

2 Model

The economy is a production economy with aggregate shocks in which agents face different employment histories and partially insure themselves through (dis)saving in capital. An inequality constraint prevents agents from borrowing, i.e., taking short positions in capital. The model is identical to the benchmark model in Krusell and Smith (1998), except that we introduce unemployment benefits; without unemployment benefits the borrowing constraint would never be binding.

Problem for the individual agent. The economy consists of a unit mass of ex ante identical households. Each period, agents face an idiosyncratic shock ε that determines whether they are employed, $\varepsilon = 1$, or unemployed, $\varepsilon = 0$. An employed agent earns a wage rate of w_t and an after-tax wage rate of $(1 - \tau_t)w_t$. An unemployed agent receives unemployment benefits μw_t. Note that Krusell and Smith set μ equal to zero. This is the only difference with their model. Markets are incomplete and the only investment available is capital accumulation. The net rate of return on this investment is equal to $r_t - \delta$, where r_t is the rental rate and δ is the depreciation rate. Agent's i maximization problem is as follows:

$$\max_{\{c_t^i, k_{t+1}^i\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^i\right)^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

$$\text{s.t.} \quad c_t^i + k_{t+1}^i = r_t k_t^i + \left[(1 - \tau_t)\bar{l}\varepsilon_t^i + \mu(1 - \varepsilon_t^i)\right] w_t + (1 - \delta)k_t^i \tag{2}$$

$$k_{t+1}^i \geq 0 \tag{3}$$

Here c_t^i is the individual level of consumption, k_t^i is the agent's beginning-of-period capital, and $\bar{l}$ is the time endowment.

Firm problem. Markets are competitive and the production technology of the firm is characterized by a Cobb-Douglas production function. Consequently, firm heterogeneity is not an issue. Let K_t and L_t stand for per capita capital and the employment rate, respectively. Per capita output is given by

$$Y_t = a_t K_t^{\alpha} (\bar{l} L_t)^{1-\alpha} \tag{4}$$

and prices by

$$w_t = (1-\alpha)\, a_t \left(\frac{K_t}{\bar{l} L_t} \right)^{\alpha} \text{ and} \tag{5}$$

$$r_t = \alpha a_t \left(\frac{K_t}{\bar{l} L_t} \right)^{\alpha - 1}. \tag{6}$$

Aggregate productivity, a_t, is an exogenous stochastic process that can take on two values, $1 - \Delta_a$ and $1 + \Delta_a$.

Government The only role of the government is to tax employed agents and to redistribute funds to the unemployed. We assume that the government's budget is balanced each period. This implies that the tax rate is equal to

$$\tau_t = \frac{\mu u_t}{\bar{l} L_t}. \tag{7}$$

where $u_t = 1 - L_t$ denotes the unemployment rate in period t.

Exogenous driving processes. There are two stochastic driving processes. The first is aggregate productivity and the second is the employment status. Both are assumed to be first-order Markov processes. We let $\pi_{aa'\varepsilon\varepsilon'}$ stand for the probability that $a_{t+1} = a'$ and $\varepsilon_{t+1}^i = \varepsilon'$ when $a_t = a'$ and $\varepsilon_t^i = \varepsilon'$. These transition probabilities are chosen such that the unemployment rate is a function of a only and can, thus, take on only two values. That is, $u_t = u(a_t)$ with $u_b = u(1 - \Delta_a) > u_g = u(1 + \Delta_a)$.

Parameter values Two sets of parameter values are considered. In the first economy, there is no aggregate uncertainty and in the second there is. The parameter values of the second economy correspond to those of Krusell and Smith (1998), except that the unemployed receive unemployment benefits. Its values are reported in Tables 1 and 2. The discount rate, coefficient of relative risk aversion, share of capital in GDP, and the depreciation rate take on standard values. Unemployed people receive a fixed fraction of

15% of the wage of the employed. The value of Δ_a is equal to 0.01 so that productivity in a boom, $1+\Delta_a$, is two percent above the value of productivity in a recession, $1-\Delta_a$. Business cycles are symmetric and the expected duration of staying in the same regime is eight quarters. The unemployment rate in a boom, u_g, is equal to 4% and the unemployment rate in a recession, u_b, is equal to 10%. The time endowment, $\bar{l}$, is chosen to normalize total labor supply in the recession to one. The average unemployment duration is 2.5 quarters conditional on staying in a recession and equal to 1.5 quarters conditional on staying in a boom. These features correspond with the transition probabilities reported in Table 2.

The parameter values of the economy without aggregate uncertainty are identical to those of the economy with aggregate uncertainty with the following exceptions. Δ_a is set equal to zero and the aggregate capital stock is held constant at 43. The unemployment rate is always equal to μ_b and the transition probabilities are given in Table 3.

References

Krusell, P., and A. A. Smith, Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

Table 1: Parameter Values

Parameters	β	γ	α	δ	$\bar{l}$	μ	Δ^a
Values	0.99	1	0.36	0.025	1/0.9	0.15	0.01

Table 2: Transition probabilities

s,ε / s',ε'	1-$\Delta^a, 0$	1-$\Delta^a, 1$	1+$\Delta^a, 0$	1+$\Delta^a, 1$
1-$\Delta^a, 0$	0.525	0.35	0.03125	0.09375
1-$\Delta^a, 1$	0.038889	0.836111	0.002083	0.122917
1+$\Delta^a, 0$	0.09375	0.03125	0.291667	0.583333
1+$\Delta^a, 1$	0.009115	0.115885	0.024306	0.850694

Table 3: Transition probabilities (no aggregate uncertainty)

ε / ε'	0	1
0	0.6	0.4
1	0.044445	0.955555