SHOCKS AND THE UNAVOIDABLE ROAD TO HIGHER TAXES AND HIGHER UNEMPLOYMENT

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Abstract

This paper considers a matching model in which multiple steady-state unemployment rates exist if government expenditures and unemployment benefits are high enough. The focus on the extensive margin and a possible transition to a steady state with higher unemployment rates imply that the effect of tax rates can be high even when the elasticity between consumption and leisure is low. The matching friction limits transitions between steady states due to self-fulfilling expectations. After a sufficiently large increase in the unemployment rate and after a large enough increase in the tax burden caused by an exogenous increase in government spending, however, transition towards the high-unemployment steady state is unavoidable in an economy with generous unemployment benefits.

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1 Introduction

The idea that tax rates are quantitatively important for employment has recently received a lot of attention.[1] Typically a representative agent framework is used in which the focus is on the number of hours worked. In this paper, I analyze the effect of taxes in a search model in which adjustment occurs at the extensive margin.[2] In search models, the response of a worker to a change in the tax rate depends crucially on the surplus value of working, i.e., the difference between the value of working and the value of not working. Key elements of the model developed in this paper are the following. First, the paper has low-skilled and high-skilled workers and the surplus value for the low-skilled workers is so low that changes in the tax rate can alter its sign. Second, the tax rate is endogenous, but the unemployment benefits an unemployed worker receives as well as other government expenditures are fixed. This leads to a feedback mechanism between employment and tax rates. That is, a reduction in employment decreases the aggregate tax base and increases unemployment transfers. This leads to an increase in current and/or future tax rates, which in turn lowers the surplus value of working and, thus, decreases employment.[3]

The analysis of taxes in this framework leads to several insights. One important insight of the analysis is that the *aggregate* employment response to taxes is not dictated by the elasticity of substitution between consumption and leisure of a representative agent, but depends on the cross-sectional distribution of the surplus of working versus not working. If the mass of workers with low-surplus values is large, then small changes in tax rates will have large effects, even if the change in the tax rate has no effect on the employment

[1] See, for example, Daveri and Tabellini (2000), Ohanian, Raffo, and Rogerson (2006), Prescott (2003), and Rogerson (2006).

[2] Millard and Mortensen (1997), Pissarides (1998), and Mortensen and Pissarides (2003) also study the effect of taxes in a search model. These papers treat the tax rate as an exogenous variable, whereas the endogeneity of tax rates plays a key role in this paper.

[3] Allowing for this feedback channel is important, since several European countries spend substantial amounts of money on transfers and labor market policies to reduce unemployment. Macfarlan and Oxley (1996) presents unemployment benefits paid out as a fraction of GDP in 1992 and reports numbers as high as 3.6% for Denmark. This does not include the cost of labor market policies to reduce unemployment. Also, Nickell and Ours (2000) argue that some unemployed are misclassified as sick or disabled. Finally, high unemployment can be used as an "excuse" for politicians to increase government expenditures that are not directly related to the labor market.

situation of workers with high-surplus values.[4] The differential response between agents is consistent with the empirical evidence that labor supply elasticities are low for most workers but large for some workers.[5] An important implication of this insight is that empirical studies that do not control for the cross-sectional distribution of surplus values are likely to lead to misleading estimates of the effect of tax rates on employment. A second insight of this paper's framework is that the feedback mechanism between tax rates and employment can lead to multiple steady states, which—because of the matching friction—can all be stable.[6]

The existence of multiple steady states does not necessarily imply that there are multiple equilibrium time paths and an important part of this paper is devoted to the analysis of transition dynamics. If the unemployment rate is close enough to the value of the unemployment rate in the low (high) unemployment steady state, then the economy will converge towards the low (high) unemployment steady state. For intermediate values, history does not pin down the equilibrium time path and self-fulfilling expectations are possible. Consequently, if a one-time shock leads to an unemployment rate close enough to the high-unemployment steady state, then the economy cannot move back towards the low-unemployment steady state and has to move towards the high-unemployment steady state. The reason is that even if the economy moves back towards the low-unemployment steady state, the government has to pay unemployment benefits along the transition path and this keeps the tax burden high.

The possible transition to the high-unemployment steady state implies that a one-time shock can have a large and persistent effect on unemployment, although after the shock all parameters and exogenous variables are still equal to their pre-shock values. The framework can also explain why the effect of an increase in the tax rate caused by an exogenous

[4] Chang and Kim (2006) and Gourio and Noual (2006) also point out that the aggregate elasticity of labor supply depends on the cross-sectional distribution of individual elasticities.

[5] Large elasticities have been found for low-income earners, older workers, and those considering entering the labor force. In contrast, for middle-aged men the elasticitiy has even been found to be negative. See Disney (2000) and Pissarides (1998) for a discussion.

[6] Blanchard and Summers (1987), Diamond (1982), Saint-Paul (1995), and Ortigueira (2002) also develop models with multiple steady states with different unemployment rates.

increase in government expenditures is high. First, the interaction between transfers to the unemployed and tax rates further increases the effect of the increase in government expenditures. Second, the presence of multiple steady states means that a small increase in government expenditures can generate a gradual and persistent deterioration of the labor market if it triggers the transition towards the high-unemployment steady state.

The paper is organized as follows. In Section 2, the model is described. In Section 3, transitions towards the steady states after shocks are considered. In Section 4, the differences between the balanced-budget fiscal policy and the case where the government is allowed to borrow are discussed. The last section concludes.

2 Model

The model is a very simple and stylized search model. Skill levels can take on only two values. The discrete support limits the set of possible time paths one has to consider and, thus, helps to highlight the main ideas of the paper.

2.1 Market production

There are low-productivity and high-productivity workers. The skill level of a worker is given at birth and fixed throughout the worker's life. High-skilled workers produce z_h and low-skilled worker produce z_l, with $z_l < z_h$. Market income in period t is taxed at rate τ_t. There is a unit mass of workers and the fraction of workers with productivity level k is given by $\phi_k, k \in \{l, h\}$, with $\phi_l + \phi_h = 1$.

A worker with skill level k may experience an *exogenous separation* that occurs with probability ρ_k^x. Exogenous separations reflect events that permanently destroy the productivity of a job, e.g., market conditions may shift adversely. Exogenous separations cannot occur in the period that a job is newly formed. At the beginning of each period the worker decides whether to continue in his current job. If the worker discontinues his job, he enters the unemployment pool where he searches for a new job. Workers are also subject to shocks that induce retirement, occurring at the end of a period. Let ρ^r denote the probability of retirement. A retiring agent leaves the labor market and obtains a future value of zero. Each retired worker is replaced by an unemployed new born with the same skill level.

2.2 Possibilities outside the market sector

All workers that are not employed are searching for a job and receive a new job offer (at their skill level) with probability λ_k, $k \in \{l, h\}$. All unemployed workers receive a benefit b that can be interpreted as the benefit of leisure or as home production. New born workers and workers that are unemployed because of the exogenous separation shock also receive an unemployment benefit r. Workers that choose to quit receive no unemployment benefits. Unemployment benefits are taxed at rate $\psi\tau_t$. Although benefits are taxed in some countries, they are taxed at lower rates than wage income, so that $\psi < 1$.

2.3 Decision to accept a job

Unemployed workers that obtain a job offer during period $t-1$ compare the value of a job, $W_{k,t}$, with the value of not working, $U_{k,t}$. Employed workers compare $W_{k,t}$ with the value of not working without unemployment benefits, $U^*_{k,t}$. These continuation values are equal to[7]

$$W_{k,t} = (1-\tau_t)\, z_k + \beta(1-\rho^r) \left[\rho^x_k U_{k,t+1} + (1-\rho^x_k) \max\{W_{k,t+1}, U^*_{k,t+1}\}\right], \tag{1}$$

$$U_{k,t} = b + (1-\psi\tau_t) r + \beta(1-\rho^r) \left[\lambda_k \max\{W_{k,t+1}, U_{k,t+1}\}\right], \text{ and} \tag{2}$$

$$U^*_{k,t} = b + \beta(1-\rho^r) \left[\lambda_k \max\{W_{k,t+1}, U^*_{k,t+1}\}\right]. \tag{3}$$

Workers accept job offers if $W_{k,t} > U_{k,t}$, decline job offers if $W_{k,t} < U_{k,t}$, and are indifferent when $W_{k,t} = U_{k,t}$. The difference between $W_{k,t}$ and $U_{k,t}$ is defined as the surplus, $s_{k,t}$. That is,

$$s_{k,t} = W_{k,t} - U_{k,t}. \tag{4}$$

[7] The variables τ_t, $W_{k,t}$, and $U_{k,t}$ are indexed by t to indicate their dependence on the current and expected future distribution of agents over the different employment and unemployment categories.To be precise, these variables only depend on current and future tax rates, but indirectly depend on characteristics of the cross-sectional distribution because tax rates do.

In the numerical experiments, the parameters are such that unemployment is never an attractive alternative for a worker without unemployment benefits. Consequently, endogenous destruction does not occur.[8]

2.4 Fiscal policy

In this section, I assume that the government's budget is balanced period by period and the government, thus, has to use current tax revenues to finance unemployment benefits. In particular, tax rates are solved from

$$\tau_t \left[z_l e_{l,t} + z_h e_{h,t} \right] = g + (1 - \psi \tau_t) \left[r u_{l,t} + r u_{h,t} \right], \tag{5}$$

where g is the level of per capita government expenditures which is assumed fixed, $e_{k,t}$ denotes the mass of employed workers with productivity level k in period t, and $u_{k,t}$ denotes the mass of unemployed workers with skill level k. I assume that the government is passive and simply takes the current unemployment rate as given when it sets tax rates. I will refer to this policy as the *balanced-budget* fiscal policy.

Later in the paper, I will consider a fiscal policy under which the government is allowed to borrow. Tax policy matters for the quantitative results, since the model does not satisfy Ricardian equivalence. Allowing the government to borrow, however, does not affect the main ideas of this paper.

2.5 Definition of equilibrium

The following equations give the laws of motion for $u_{l,t}$, $u_{h,t}$, $e_{l,t}$, and $e_{h,t}$. The variable $I_{k,t}$ is an indicator variable that takes on a value 1 if a newly matched worker with skill level k chooses market production and 0 otherwise.

$$\begin{gathered} u_{k,t} = u_{k,t-1} \\ +\phi_k \rho^r + (1-\rho^r)\rho_k^x e_{k,t-1} - \left[\rho^r + (1-\rho^r)\lambda_k I_{k,t}\right] u_{k,t-1} \\ \text{for} \quad k \in \{l, h\}, \end{gathered} \tag{6}$$

[8] Consequently, there is no role in this model for layoff taxes. See Ljungqvist and Sargent (2006) for an analysis of a matching model with unemployment benefits and layoff taxes.

and

$$e_{k,t} = e_{k,t-1}$$
$$-[\rho^r + (1-\rho^r)\rho_k^x]e_{k,t-1} + (1-\rho^r)\lambda_k I_{k,t} u_{k,t-1} \qquad (7)$$
$$\text{for } k \in \{l, h\}.$$

An equilibrium time path is a set of values for $I_{k,t}$ such that (i) unemployment and employment levels, $u_{k,t}$ and $e_{k,t}$, are determined by (6) and (7), (ii) tax rates, τ_t, are determined by (5), (iii) the value of working, $W_{k,t}$, and the values of not working, $U_{k,t}$ and $U^*_{k,t}$, are solved from (1), (2), and (3) respectively, (iv) the value of $I_{k,t}$ is consistent with the relative ranking of $W_{k,t}$ and $U_{k,t}$, and (v) $W_{k,t} > U^*_{k,t}$.

2.6 Multiple Steady states

The two obvious steady states to consider are the following. In the first steady state, $I_k = 1$ and $W_k > U_k$ for both values of k. That is, all workers prefer market production over the non-market alternative. In this steady state, only the newly born and those that have experienced an exogenous separation are unemployed. Let u_l^{low} be the mass of low-skilled unemployed in this steady state. In the second steady state, it still is the case that $I_h = 1$ and $W_h > U_h$, but now $I_l = 0$ and $W_l < U_l$, resulting in all workers with low productivity levels being unemployed (in addition to the workers with high skills that have either experienced an exogenous breakup or are newly born). Let u_l^{high} be the mass of low-skilled unemployed in this steady state.

The intuition for the possible existence of multiple steady states is straightforward. When the unemployment rate is high, the tax base is low while the amount of unemployment benefits the government has to pay out is high. Consequently tax rates are high resulting in a low surplus, which means that indeed more workers decide not to participate in market activity. Analogously, there is a steady state with a low unemployment rate.[9]

It is easy to find parameter values such that both steady state solutions exist. Tax rates must be positive, so either g or r must be positive. Also, market production must be

[9] Both the tax rate and total tax revenues are higher in the high-unemployment steady state. That is, the economy as a whole is on the upward sloping part of the Laffer curve. Tax revenues from low-skilled workers are, of course, lower in the high-unemployment steady state. So this part of the economy operates on the downward sloping part.

taxed at a higher rate than non-market activities, thus, either $b > 0$ or $\psi < 1$ (or both).[10] If these two conditions are met, then it is easy to find parameters such that the benefits of being unemployed, $b + (1 - \tau_r)r$, are close to the productivity level of the low-skilled worker, z_l. This will ensure that there are marginal workers. The parameters in this paper are such that these conditions are satisfied so that both a low and a high-unemployment steady state exist.

One can also construct steady states in which agents of the same skill level have a different employment status. This would, of course, require that $W_k = U_k$ for either $k = l$ or $k = h$. To see how this can be done, consider the steady state with $W_l < U_l$ and $W_h > U_h$. Now assume that a fraction ϕ^* of high-skilled unemployed workers does not accept job offers. If this fraction increases then the tax rate increases and the surplus $W_h - U_h$ decreases and clearly will become negative before ϕ^* has reached a value equal to one. Consequently, there is a value of ϕ^* such that $W_h = U_h$ and this corresponds to a steady state equilibrium. This type of steady state will not be considered in this paper.

2.7 Uniqueness of Equilibrium

In this section, I discuss how to check whether an equilibrium time path is unique and for which range of values for the initial unemployment rate it is unique. In the initial time period, t_0, the mass of low-skilled unemployed workers, u_{l,t_0}, is assumed to be in between the two steady state values: u_l^{low} and u_l^{high}. The mass of high-skilled unemployed workers is in the first period set equal to its steady state value, $u_{h,t_0} = u_h^{low} = u_h^{high}$.

Condition A. *Parameter values are such that the low-unemployment steady state, with $W_l > U_l$ and $W_h > U_h$, as well as the high-unemployment steady state, with $W_l < U_l$, $W_l > U_l^*$, and $W_h > U_h$, exist.*

Condition B. $0 < \lambda < 1 - \rho^x < 1$.

It is easy to find parameter values such that condition A is satisfied. In particular, one can choose b such that the surplus value of the low-skilled workers in the low-unemployment

[10] If $b = 0$ and $\psi = 1$, then the surplus does not depend on any aggregate variable.

steady state is barely positive. If the mass of low-skilled workers is positive, then the higher tax rate of the high-unemployment steady state will make the surplus negative. Condition B is a regularity condition that says that an employed worker is more likely to be employed next period than an unemployed worker and there is always some unemployment because $\rho^x > 0$.

Definition of $\Im^{low}$. This is the time path corresponding to $I_{l,t} = I_{h,t} = 1$, that is, the time path along which the economy moves towards (or remains in) the low-unemployment steady state.

Definition of $\Im^{high}$. This is the time path corresponding to $1 - I_{l,t} = I_{h,t} = 1$, that is, the time path along which the economy moves towards (or remains in) the high-unemployment steady state.

Lemma 1. *Assume that Conditions A and B hold. If $u_l^{low} < u_{l,t_0} < u_l^{high}$, then*

- $\tau_{t+1} < \tau_t$ *and* $W_{k,t+1} - U_{k,t+1} = s_{k,t+1} > W_{k,t} - U_{k,t} = s_{k,t}$ *along* $\Im^{low}$
- $\tau_{t+1} > \tau_t$ *and* $W_{k,t+1} - U_{k,t+1} = s_{k,t+1} < W_{k,t} - U_{k,t} = s_{k,t}$ *along* $\Im^{high}$

Proposition 1. *Assume that Conditions A and B hold and $u_l^{low} \leq u_{l,t_0} \leq u_l^{high}$.*

- *If $\Im^{low}$ ($\Im^{high}$) is not an equilibrium then $\Im^{high}$ ($\Im^{low}$) is an equilibrium.*
- *If and only if $\Im^{low}$ ($\Im^{high}$) is not an equilibrium then $\Im^{high}$ ($\Im^{low}$) is the unique equilibrium.*

Proposition 2. *Assume that Conditions A and B hold. If the two steady states are both unique continuation equilibria, then there are values $\overline{u}_l^{low}$ and $\overline{u}_l^{high}$ so that*

- $u_l^{low} < \overline{u}_l^{low} < \overline{u}_l^{high} < u_l^{high}$,
- $\Im^{low}$ *is the unique equilibrium if* $u_{l,t_0} \in [u_l^{low}, \overline{u}_l^{low})$,
- *multiple equilibrium time paths are possible if* $u_{l,t_0} \in [\overline{u}_l^{low}, \overline{u}_l^{high}]$, *and*

- $\Im^{high}$ *is the unique equilibrium if* $u_{l,t_0} \in (\overline{u}_l^{high}, u_l^{high}]$.

The lemma shows that economic conditions strictly improve (worsen) along the transition path towards the low(high)-unemployment steady state. The first part of Proposition 1 shows that the economy can always converge towards at least one of the two steady states. The second part shows that to check whether an equilibrium time path towards a steady state is unique, one only has to check whether the time path towards the other steady state is an equilibrium. If it is not, then the equilibrium time path is unique.

The proofs are given in the appendix, but the intuition for the results is straightforward. The economy can always move towards one steady state for the following reason. Along the path towards the low(high)-unemployment steady state, the unemployment rates and thus the tax rates are strictly de(in)creasing, which in turn implies that the surplus values are strictly in(de)creasing. For example, suppose that the economy cannot move towards the low-unemployment steady state. This means that initially the surplus is negative, despite the bright future facing the workers. But this means that the surplus would initially be even lower if workers would face the worse path towards the high-unemployment steady state. Since the surplus is decreasing along $\Im^{high}$, it would always be negative and moving towards the high-unemployment steady state would be an equilibrium.

Suppose that $\Im^{high}$ is an equilibrium. How can one verify that this is a unique equilibrium time path? Along this transition path $s_{l,t} \leq 0$. Suppose that $s_{l,t_0} = 0$ in the initial period, t_0. Then moving towards the low-unemployment steady is also an equilibrium,[11] so one only has to consider the case with $s_{l,t_0} < 0$. Suppose there is an equilibrium other than the one with $I_{l,t} = 0$ for all t. For this alternative equilibrium time path it must be the case that $s_{l,t} \geq 0$ for some t. The time path for which that is most likely to happen is the time path with the lowest tax rates. But this is the time path that moves directly towards the low-unemployment steady state, i.e., $\Im^{low}$. If even this time path is not an equilibrium, then moving towards the high-unemployment steady state is a unique equilibrium. This result is very helpful computationally, because it means that to check whether

[11] If the economy moves towards the low-unemployment steady state, then the surplus in period t_0 cannot be lower so it cannot be negative. Since the surplus is increasing along $\Im^{low}$, the surplus is always positive.

multiple equilibrium time paths exist for an initial unemployment rate, one only has to check at most two possible time paths.

To understand Proposition 2, note that it must be the case that $s_{l,t_0} > 0$ when $u_{t_0} = u_l^{low}$ and $\Im^{high}$ is being followed in the future. With $s_{l,t_0} > 0$, $\Im^{high}$ is not an equilibrium. If $s_{l,t_0} \leq 0$, then the low-unemployment steady state would not be a unique continuation equilibrium. Since s_{l,t_0} is strictly positive it will remain so if the starting situation is made slightly worse, that is, if u_{t_0} slightly increases. For these values of u_{t_0} the time path $\Im^{high}$ is, thus, also not an equilibrium. Consequently, $\Im^{low}$ is the unique equilibrium. As u_{t_0} increases the surplus values decrease and the value of u_{t_0} for which s_{l,t_0} turns zero if $\Im^{high}$ is followed is the value of $\overline{u}_l^{low}$.

If the low-unemployment steady state is not a unique continuation equilibrium, then $\overline{u}_l^{low} = u_l^{low}$ and the two relevant regions are $[u_l^{low}, \overline{u}_l^{high}]$ and $(\overline{u}_l^{high}, u_l^{high}]$. Similarly, $\overline{u}_l^{high} = u_l^{high}$ if the high-unemployment steady state is not a unique continuation equilibrium. If neither is a unique continuation equilibrium, then multiple equilibria are possible for any initial unemployment rate in $[u_l^{low}, u_l^{high}]$.

History determines what the current unemployment rate is. The proposition shows that for some values this uniquely determines the equilibrium, namely those below $\overline{u}_l^{low}$ and above $\overline{u}_l^{high}$ but that for others self-fulfilling expectations are possible.[12]

2.8 Computation of Equilibrium

To check whether the time paths towards the steady states, i.e., $\Im^{low}$ and $\Im^{high}$, are equilibrium time paths, I use a simple computational procedure. Note that along these time paths the values of $u_{k,t}$ and $e_{k,t}$ converge towards their steady state values, which in turn implies that the tax rate, τ_t converge. This implies that the continuation values converge towards the steady state values corresponding to the limiting values of $I_{k,t}$. Calculation of steady state values is straight forward. To calculate the continuation values along the transition path one first chooses a T such that for $t > T$ the distance between τ_t and the steady state value is less than a specified small number. One then sets $W_{k,T}$

[12] The role of history versus expectations is analyzed in different contexts in by Krugman (1991), Matsuyama (1991), and Zilibotti (1995).

and $U_{k,T}$ equal to the steady state values. Using $W_{k,T}$ and $U_{k,T}$ as "initial" values one can then use Equations (1) and (2) to calculate values of $W_{k,t}$ and $U_{k,t}$ for $t < T$. If the relative ranking of $W_{k,t}$ and $U_{k,t}$ corresponds with the value of $I_{k,t}$ for all k, then this is an equilibrium.

3 Shocks

In this model, agents face idiosyncratic uncertainty regarding exogenous separations, retirement, and obtaining job offers. This does not generate aggregate uncertainty, however, since there is a continuum of agents. In this section, I discuss how the economy responds to aggregate shocks. In the main experiment, a one-time burst of separations increases the unemployment rate, but all parameter values are back to their pre-shock values in the subsequent period. I also consider a permanent increase in government expenditures. To keep the analysis tractable, I will retain the perfect foresight feature of the model. That is, agents think shocks to the system cannot happen and after the shock put again zero probability on such an event.

3.1 Parameter values

The objective of this section is to illustrate that the model can generate remarkable dynamics that standard models without multiple steady states cannot generate. This section highlights two properties in particular. First, it provides a numerical example of an economy that cannot recover after a large enough one-time shock and instead remains in a situation with high unemployment rates. This property is a direct consequence of Proposition 2. Second, it documents that an increase in the tax burden that is triggered by a small increase in government expenditures can generate a slow but steady increase in the unemployment rate that continues until the economy has reached the new steady state. These experiments make clear that this framework with transitions from one steady state to another provides a powerful magnification mechanism.

The magnitude of the generated increase in unemployment rates depends on the mass of marginal workers. If the mass of marginal workers is high then small changes can have large effects. In the numerical example, I generate a substantial increase in the unemployment rate by (i) setting the benefits of being unemployed relative to the productivity of the low-skilled worker high enough so that low-skilled workers are indeed marginal workers and (ii) setting the mass of low-skilled workers high enough. A proper evaluation of the channel called into attention in this paper would require using a finer distribution of skill levels and allowing for differences in the benefits of not working across workers. Calibrating such a model would be difficult. Moreover, the advantage of the discrete support used here is that one only has to consider two steady states and two time paths. In a richer environment the analysis is likely to become quickly intractably.

Parameter values are given in Table 1. Values of tax rates, unemployment rates, and unemployment benefits as a fraction of output are given in Table 2. The unemployment rate is equal to 3.9% in the low unemployment steady state and equal to 10.3% in the high unemployment steady state. The tax burden (total government outlays over output) is equal to 32.2% in the low-unemployment steady state and equal to 36.8% in the high-unemployment steady state. There are two reasons for the increase in the tax rates. First, unemployment benefits paid out increase with 2.8 percentage points from 1.5% of GDP to 4.3% of GDP. The remaining 1.8 percentage point increase in the tax burden is due to the decrease in the tax base so that the workers that remain working have to pay a larger fraction in taxes to finance government outlays.

3.2 One-time shocks in the low-unemployment steady state

The numerical example of this section illustrates possible time paths after an exogenous increase in the mass of unemployed low-skilled workers.[13] After this one-time event, all parameter values are again equal to their pre-shock values. The increase could have been caused, for example, by a one-time change in the rate of exogenous break-ups.[14] All that matters, however, is the resulting increase in the unemployment rate and not the cause

[13] With one-time changes in aggregate productivity the possible magnitudes of the response of the unemployment rate are limited because of the limited amount of heterogeneity. This is not the case with a shock in the exogenous separation rate.

[14] There is some evidence that separations did increase during the seventies. For example, Nickell, Layard, and Jackman (2005) report that the separation rate increased from a low of 1.8% in 1969 to a high of 3.4% in 1982.

for it.

Suppose that the low-unemployment steady state is a unique continuation equilibrium. Thus, $\overline{u}_l^{low} > u_l^{low}$. If the shock is small enough and the unemployment rate stays below $\overline{u}_l^{low}$, then it is not possible that the economy will converge towards the high-unemployment steady state. Instead, the unemployment rate will decrease and the economy will converge back towards the low-unemployment steady state.

Clearly, there are large enough shocks such that the economy will end up in the high-unemployment steady state. In particular, if the one-time increase in the exogenous separation rate brings about the unemployment of all low-skilled workers then the economy moves to the high-unemployment steady state instantaneously. If the high-unemployment steady state is a unique continuation equilibrium, then $\overline{u}_l^{high} < u_l^{high}$. Consequently, shocks that lead to unemployment rates close enough to the level of the high-unemployment regime cause an inevitable transition to the high-unemployment steady state .

Multiple equilibrium time paths are possible for intermediate shocks. Figure 1 graphically displays the possible time paths for different size shocks using the parameter values of Table 1. In particular, it plots the time path for the unemployment rate after the largest possible increase in the unemployment rate such that it is still possible to move back to the low-unemployment steady state. In addition, it plots the unemployment rate towards the high-unemployment steady state for the smallest shock such that convergence towards the high-unemployment steady state is still possible. For these parameter values the unemployment rate has to increase to a level above 8.7% for the time path back towards the low-unemployment steady state not to be an equilibrium time path.

3.3 One-time shocks when institutions are better

The analysis above showed that for a large enough shock unemployment rates cannot come down towards pre-shock levels. Here parameter values are such that the high-unemployment steady state exists and is a unique continuation equilibrium. For lower unemployment benefits or government expenditures the high-unemployment steady state would not be a unique continuation equilibrium. This means that the economy could move towards the high-unemployment state after a large enough shock, but also could move back (as long as $u_{l,t_0} \leq u_l^{high}$). If unemployment benefits or government expenditures are so low enough that the high-unemployment steady state does not exist, then the economy has to converge towards the low-unemployment steady state.

3.4 Permanent increases in government expenditures

In this section, I consider a permanent increase in government expenditures, g, and discuss possible responses. Suppose that g is low enough so that the high-unemployment steady state does not exist. When all other parameters are kept equal to the values given in Table 1, then this would occur if $g < 0.325$. To interpret numerical values, I define $\overline{g}$ as the value of government expenditures as a fraction of total output in the low-employment steady state. For $g = 0.325$, this gives $\overline{g} = 28.5\%$.For $\overline{g} = 28.5\%$ the high-unemployment steady state exists. For that value of g, the low-unemployment steady state is a unique continuation equilibrium, so the economy could not go to the high-unemployment steady state, unless there is a shock to the system. When $\overline{g}$ rises above 31.2%, the low-unemployment steady state is no longer a unique continuation equilibrium and a transition to the high-unemployment steady state because of self-fulfilling expectations is a possibility. Finally, as $\overline{g}$ rises above 32.5%, the low-unemployment steady state no longer exists. Consequently, if the economy starts out in the low-unemployment steady state, then a small change in g, from a value just below 32.5% to a value just above 32.5%, will trigger an unavoidable transition to the high-unemployment steady state.

In this model, workers do not quit because they are not entitled to benefits after a quit. Consequently, the higher tax rates only affect the job acceptance decision. This means that the transition to the high-unemployment steady state is a slow and gradual process.

3.5 Shocks in the high-unemployment steady state

Above, it was shown that a one-time shock could put the economy on a path towards the high-unemployment steady state when the economy started in the low-unemployment steady state. Because of the matching friction, it is possible that no one-time shock of any size can put an economy that starts out in the high-unemployment steady state on a path towards the low-unemployment steady state.

The reason is that the matching friction limits the number of workers that can start a job. This is true even if during one period conditions are so good that even the least productive worker would accept a job offer. Consequently, the reduction in the unemployment rate is limited. After the maximum possible reduction, the level of the unemployment rate may still be too high to make convergence towards the low-unemployment steady state possible. This is the case for the parameters given in Table 1. In particular, all job offers must be accepted for at least fourteen periods before the unemployment rate has dropped to a level at which the economy could converge towards the low-unemployment steady state.

4 Tax now versus tax later

The purpose of this section is threefold. First, I want to point out that qualitatively the results of this section do not change if the government is allowed to borrow. Second, although the results do not change qualitatively, they do change quantitatively. Consequently, the particular type of fiscal policy followed can have important consequences. Third, I show that—in contrast to the results shown in the literature—allowing the government to deviate from the balanced-budget fiscal policy makes it more likely that multiple equilibrium time paths exist.

Note that the model does not satisfy the Ricardian equivalence property, since an employed worker would rather be taxed in the future when he may face lower tax rates (either because he is unemployed or because he is retired). Besides allowing the government to borrow, there are many other interesting tax policies that one could consider. For example, instead of subsidizing the unemployed one could subsidize low-skilled workers either by explicit subsidies or lower tax rates. These alternatives are not considered here.[15]

[15] Also, we do not consider the case where the government first raises tax rates on the (secure) high-skilled workers until it has build up enough funds to finance the transition towards the low-unemployment steady state.

Balanced-NPV fiscal policy The balanced-NPV fiscal policy is characterized by a *constant* tax rate for which the net present value of government revenues is equal to the net present value of government expenditures.[16] This constant tax rate, τ, satisfies

$$\tau \sum_{j=1}^{\infty} \beta^{j-1} \left[z_l e_{l,j} + z_h e_{h,j} \right] = \sum_{j=1}^{\infty} \beta^{j-1} \left[g + r u_{l,t} + r u_{h,t} \right] - \psi \tau \sum_{j=1}^{\infty} \beta_t^{j-1} \left[r u_{l,t} + r u_{h,t} \right]. \quad (8)$$

Multiplicity less likely with a balanced-budget policy In the numerical example discussed above, staying in the high-unemployment steady state is the unique continuation equilibrium and moving towards the low unemployment steady state is not possible. Allowing the government to borrow, i.e., shift the tax burden into the future, makes it more likely that the economy could move towards the low unemployment steady state. The reason is that market production is taxed at a higher rate than non-market benefits, which means that shifting the tax burden into the future increases the value of working versus not working (since only with some probability is a working agent still working when the higher future tax burden occurs).

In fact, for the parameters considered in Table 1, the high-unemployment steady state is not a unique continuation equilibrium under the balanced-NPV fiscal policy. Of course, this depends crucially on parameter values. When b increases to a value above 0.268 then the surplus in the high-unemployment steady state is so low that it is a unique continuation equilibrium under the balanced-NPV fiscal policy as well.

Fiscal policy and multiplicity in the literature In a classic paper, Schmitt-Grohé and Uribe (1997) argue against a balanced budget-fiscal policy by showing that it can make expectations of higher (lower) tax rates self-fulfilling. In contrast, in this paper expectations of higher or lower tax rates are more likely to be self-fulfilling with a balanced-

[16] There are other fiscal policies in the set that satisfy the government's intertemporal budget constraint. However, if the direct transition towards the low-unemployment steady state is not an equilibrium under the policy with a constant tax rate, then it is not an equilibrium for any policy in the set of feasible policies. To see why suppose that transition with a constant tax rate is not an equilibrium. This means that the (constant) surplus value of low-skilled jobs is negative. Lowering the tax rate can turn the surplus positive in some periods, but to satisfy the intertemporal budget constraint, tax rates must be increased at some other point to compensate. For those periods, the already negative surplus becomes even more negative. Since the fiscal policy with a contant tax rate is the one most likely to get the economy out of the high-unemployment steady state, this is the most interesting one to consider.

NPV fiscal policy than with a balanced-budget fiscal policy. The reason for the difference is that Schmitt-Grohé and Uribe (1997) consider indeterminacies around one unique steady state, whereas I consider different steady states. In this paper, a balanced-NPV fiscal policy reinforces the expectations of the private sector and, thus, makes them more likely to become true. This would be undesirable when the economy is in the low-unemployment steady state, but would be desirable if the economy is in the high-unemployment steady state.

5 Concluding comments

The interaction between the acceptance decision and the level of the tax rate is an essential part of this paper's model. This leads to the existence of multiple steady states. The existence of multiple steady states in turn implies that relatively small changes can have large effects if they trigger a transition from one steady state to another. The matching friction is also an essential ingredient. It leads to the possibility that the steady states are unique continuation equilibria and that only shocks, such as an increase in the destruction rate, or structural changes in the economy, such as an increase in government spending, can initiate a movement towards the other steady state.

In the numerical example of this paper, the large increase in the unemployment rate is driven by the assumption that the mass of low-skilled workers is high and that for them the value of working is close to the value of not working. The existence of multiple steady states, however, does not depend on the assumption of a large mass of low-skilled workers. In fact, it is possible to have a large number of skill levels, each with a small number of agents, and also many steady states. Whenever workers of a particular skill level become unemployed, then the tax rate increases, which puts the group of worker with the next skill level at risk.[17]

To document that this mechanism is an important part behind the persistent rise in European unemployment rates would require a calibration of not only the distribution of skill levels but also of the distribution of outside options and would, for example, take into

[17] A simple example of such an economy is given in Appendix B.

account aspects such as age, family composition, and retirement possibilities. This clearly would not be an easy task.[18] Also, the analysis would quickly become intractable when the cross-sectional distribution of the model would become more complex.

It seems unlikely, that the *endogenous* increase in the tax rates emphasized in this paper is the dominant force behind the increase in the European unemployment rate. There are two reasons for this. First, if endogenous increases in the tax rates explain all changes in unemployment rates, then increases in the tax rates caused by increases in government expenditures not related to the increase in the unemployment rate would have no effect. This clearly would be inconsistent with the spirit of the paper, although one could argue that a deterioration of the labor market made it possible for politicians to increase government spending, so that such an increase in government spending is also an endogenous consequence of the higher unemployment rates.

The second reason why pure endogenous increases in tax rates cannot be the complete story is that—although unemployment rates displayed a sharp increase in the seventies—employment rates started to decline before the seventies, i.e., before a major shock could trigger an endogenous increase in the tax rate.

Although the channel emphasized here cannot stand on it own, it can help to understand why a steady exogenous increase in government outlays and, thus, the tax burden can have such a large effect. First, the emphasis on the extensive margin avoids the inconsistency with micro studies that show that the labor supply elasticity for many workers is not high. Second, with a steady increase in the fraction of GDP allocated to government expenditures, the mass of workers affected is bound to be large at some point. Third, the presence of multiple steady states can magnify increases in government expenditures if they trigger a transition to a steady state with a higher unemployment rate. The transition may not be as large in the numerical example used in this paper, but even transitions from steady states with more similar unemployment rates magnify the response in the

[18] Chang and Kim (2006)make an important first step by calibrating earnings ability. Their formulation allows for limited heterogeneity (between males and females) and does not allow for different outside opportunities for, for example, the young, the old, and single-parent households. Their framework may, thus, very well underestimate the mass of marginal jobs. Nevertheless, even with this limited amount of heterogeneity, they already establish that the aggregate labor supply elasticitiy is substantially above the labor supply elasticity of the individual.

unemployment rate.

A Proofs

Proof of Lemma 1. Consider $\Im^{low}$, i.e., the path with $I_{l,t} = I_{h,t} = 1$. If $u_{l,t} = u_l^{low}$, then the flow of low-skilled workers into unemployment is equal to the flow out of unemployment. If $u_{l,t} > u_l^{low}$, then the flow out of unemployment will be higher than the flow into unemployment, which means that $u_{l,t+1} < u_{l,t}$. This implies that tax rates are strictly decreasing when either $r > 0$ or $g > 0$. The decrease in unemployment rates would not imply a reduction in tax rates when $r = gov = 0$, but for these parameter values it would not be possible that $u_l^{low} < u_l^{high}$. Along $\Im^{high}$ the unemployment rate and tax rates are clearly increasing unless $\rho^x = \rho^r = 0$, which is ruled out by Condition B.

It remains to be shown that the surplus is monotonically de(in)creasing along $\Im^{high}$ ($\Im^{low}$). Consider $\Im^{high}$. Let $s_{l,t}^{high}(u_{l,t_0})$ be the surplus of the low-skilled workers along $\Im^{high}$ when the *initial* unemployment rate for the low skilled is u_{l,t_0}. It is equal to

$$\begin{gathered} s_{l,t}^{high}(u_{l,t_0}) = \eta_t + \beta^r(1-\rho^x)s_{l,t+1}^{high}(u_{l,t_0}) - \beta^r\lambda \max\left\{0, s_{l,t+1}^{high}(u_{l,t_0})\right\} \\ \text{with } \eta_t = -\tau_t(z_l - \psi r) + z_l - b - r, \end{gathered} \tag{9}$$

where $\beta^r = \beta(1-\rho^r)$.[19] From this equation follow the following two properties.

1. If $s_{l,t}^{high}(u_{l,t_0}) \geq 0 \; \forall t$ or when $s_{l,t}^{high}(u_{l,t_0}) \leq 0 \; \forall t$, then the max operator can be replaced by either $s_{l,t}^{high}(u_{l,t_0})$ or by 0 in each period. $s_{l,t}^{high}(u_{l,t_0})$ is then simply a convergent sum and each of the elements of $s_{l,t}^{high}(u_{l,t_0})$ is smaller than the corresponding element of $s_{l,t+j}^{high}(u_{l,t_0})$ for all $j > 0$, since $\eta_t > \eta_{t+1}$.

2. Suppose $\exists T$ such that $s_{l,T}^{high}(u_{l,t_0}) > s_{l,T+1}^{high}(u_{l,t_0})$, then it follows directly from 9 that $s_{l,T-1}^{high}(u_{l,t_0}) > s_{l,T}^{high}(u_{l,t_0})$. Thus, if the inequality can be established for $t = T$, then by iterating backwards the inequality holds for all $t < T$.

[19] For a worker that remains in his job, $s_{l,t+1}^{high}$, could be negative. This does not trigger a separation, since that would require that $W_{l,t+1} < U_{l,t+1}^*$, which would violate Condition A.

To prove the lemma, consider the two possible cases for the sign of the surplus of the low-skilled workers in the high-unemployment steady state, $\boldsymbol{s}_l^{high}$, i.e., zero and strictly negative. The limit of $s_{l,t}^{high}(u_{l,t_0})$ as $t \longrightarrow \infty$ is equal to $\boldsymbol{s}_l^{high}$. If $\boldsymbol{s}_l^{high} < 0$ then $\exists T$ such that $s_{l,t}^{high}(u_{l,t_0}) < 0$ for $t \geq T$. According to Property #1, $s_{l,t}^{high}(u_{l,t_0})$ is decreasing with t for $t \geq T$. Because of Property #2, $s_{l,t}^{high}(u_{l,t_0})$ is also decreasing for $t < T$.

If $\boldsymbol{s}_l^{high} = 0$, then the steady state (and limiting value) of η_t is equal to zero, but his means that $\eta_t > 0 \ \forall t$, since η_t is strictly decreasing. According to Condition B, $(1 - \rho^x) > \lambda$. Thus $s_{l,t}^{high}(u_{l,t_0}) > 0 \ \forall t$, which means that Property 1 can be used.

Showing that the surplus is increasing along $\Im^{low}$ follows the same steps.

Proof of Proposition 1 - first part. The assumption that the two steady states exist implies directly that the proposition holds for $u_{l,t_0} = u_l^{low}$ and $u_{l,t_0} = u_l^{high}$. Now consider the case where $u_l^{low} < u_{l,t_0} < u_l^{high}$. Suppose that $\Im^{low}$ is not an equilibrium and suppose to the contrary that $\Im^{high}$ is not an equilibrium either. If $\Im^{low}$ is not an equilibrium, then at least one surplus value is negative along $\Im^{low}$, which according to Lemma 1 implies that $s_{l,t_0}^{low}(u_{l,t_0}) < 0$. But if $s_{l,t_0}^{low}(u_{l,t_0}) < 0$, then $s_{l,t_0}^{high}(u_{l,t_0})$ is definitely negative, since $\Im^{high}$ is less favorable than $\Im^{low}$. The surplus along $\Im^{high}$ is, thus, negative in every period, since surplus values are decreasing along $\Im^{high}$ according to Lemma 1. This means that $\Im^{high}$ is an equilibrium, which contradicts that it is not an equilibrium.

Proof of Proposition 1 - second part. Suppose that $\Im^{low}$ is not an equilibrium. From the first part of the proposition, it follows that $\Im^{high}$ is an equilibrium. The question is whether there are other equilibria, that is, whether there are time paths for which $I_{l,t} = 1$ for some (but not all) t. Key in showing that $\Im^{high}$ is unique is that $\Im^{low}$ corresponds to the time path with the lowest tax rates. Consequently, of all time paths with $I_{l,t} = 1$ for some t, $\Im^{low}$ is the one most likely to be an equilibrium.

If $\Im^{low}$ is not an equilibrium then $s_{l,t_0}^{low}(u_{l,t_0}) < 0$. Now consider other time paths that set $I_{l,t} = 1$ for some (but not all) time periods. First, consider the time path for which $I_{l,t_0} = 1$, but $I_{l,t} = 0$ for some $t > t_0$. If $s_{l,t_0} < 0$ when $I_{l,t} = 1$ for all t, then clearly $s_{l,t_0} < 0$ for a time path with higher unemployment rates and, thus, higher, tax rates further along the time path. Consequently, any time path with $I_{l,t_0} = 1$ cannot be an equilibrium time path.

Next, consider the time path for which $I_{l,t_0} = 0$, but $I_{l,t} = 1$ for some $t > t_0$. The value of the surplus of the low-skilled workers in period $t_0 + 1$ is less than $s^{low}_{l,t_0}(u_{l,t_0})$. The reason is that by setting $I_{l,t_0} = 0$, the unemployment rate has increased and all possible time paths of tax rates are uniformly dominated by the time path of tax rates under $\Im^{low}$. Thus, $I_{l,t_0} = 0$ and $I_{l,t_0+1} = 1$ cannot be an equilibrium either. Iteration on this argument gives that $I_{l,t}$ must be equal to zero for all t, that is, $\Im^{high}$ is the unique equilibrium.

The "only if" part is trivially true, since if $\Im^{high}$ is an unique equilibrium, then $\Im^{low}$ is, of course, not an equilibrium.

Proof of Proposition 2. If the low-unemployment steady state is a unique continuation equilibrium then $s^{high}_{l,t_0}(u^{low}_l) > 0$. If it would be non-positive, then—according to Lemma 1—it would be non-positive for every t and consequently $\Im^{high}$ would be an equilibrium as well.

The surplus is a continuous function of tax rates, according to Equation (9). The tax rate is a continuous function of the unemployment rate. Next period's unemployment rate is a continuous function of this period's unemployment rate. Consequently, if $s^{high}_{l,t_0}(u^{low}_l) > 0$, then it is still positive if u_{l,t_0} is slightly higher than u^{low}_l. Thus, $\Im^{high}$ is not an equilibrium for this slightly higher unemployment rate either, which means according to Proposition 1 that $\Im^{low}$ is the unique equilibrium time path.

As u_{l,t_0} increases, $s^{high}_{l,t_0}(u_{l,t_0})$ decreases. Moreover, there exists a value $\overline{u}^{low}_l$ such that $s^{high}_{l,t_0}(\overline{u}^{low}_l) = 0$, since $s^{high}_{l,t_0}(u^{high}_l) < 0$, $s^{high}_{l,t_0}(u^{low}_l) > 0$, and $s^{high}_{l,t_0}(u_{l,t_0})$ is a continuous function. For $u_{l,t_0} = \overline{u}^{low}_l$, $\Im^{high}$ is an equilibrium, but for $u_{l,t_0} < \overline{u}^{low}_l$ it is not. Thus, if $u_{l,t_0} \in [u^{low}_l, \overline{u}^{low}_l)$, then $\Im^{low}$ is the only equilibrium.

If $s^{high}_{l,t_0}(\overline{u}^{low}_l) = 0$, then $s^{low}_{l,t_0}(\overline{u}^{low}_l) > 0$, since tax rates are strictly lower along $\Im^{low}$. Consequently, for $u_{l,t_0} = \overline{u}^{low}_l$, $\Im^{low}$ is also an equilibrium time path. Using the same continuity argument as the one used above, it can be shown that $\Im^{low}$ is still an equilibrium time path for values of u_{l,t_0} slightly higher than $\overline{u}^{low}_l$. Since initial conditions deteriorate, $\Im^{high}$ will definitely remain an equilibrium. Again because of continuity, there is a value

$\overline{u}_l^{high}$ such that $s_{l,t_0}^{low}(\overline{u}_l^{high}) = 0$, which means according to Lemma 1 that $\Im^{low}$ is (just) an equilibrium time path for $u_{l,t_0} = \overline{u}_l^{high}$. Thus, if $u_{l,t_0} \in [\overline{u}_l^{low}, \overline{u}_l^{high}]$, then both $\Im^{low}$ and $\Im^{high}$ are equilibrium time paths.

Finally, if $u_{l,t_0} > \overline{u}_l^{high}$ then $s_{l,t_0}^{low}(u_{l,t_0}) < 0$, which means that $\Im^{low}$ is not an equilibrium, which means according to proposition 2 that $\Im^{high}$ is the unique equilibrium. That is, if $u_{l,t_0} \in (\overline{u}_l^{high}, u_l^{high}]$, then $\Im^{high}$ is the unique equilibrium time path.

B Multiple steady states and unraveling

In the model described in the main text, there are only two skill levels. The analysis in this section makes clear that even when workers are distributed uniformly across many skill levels a tiny initial change in the tax rate can still lead to a large change in employment through a domino effect. It is true, however, that the distribution of workers' productivity levels should be "dense enough".

To simplify the analysis, a static version of the model is used. Workers have different productivity levels, $z_1 \leq z_2 \leq \cdots \leq z_N$, where N can be arbitrarily large. At each production level is an equal mass of agents. As before, tax rates are assumed to be proportional. The surplus of working over not working is defined by

$$s_n(z_j) = (1 - \tau_n)\; z_j - b - r, \tag{10}$$

where $s_n(z_j)$ is the value of the surplus of worker with skill level j when n workers are unemployed. Note that $s_n(z_n)$ is the surplus value of the unemployed worker with the highest value for z. The tax rate when n workers are unemployed, τ_n, is given by

$$\tau_n = \frac{nr}{\sum_{i=n+1}^{N} z_j} = \frac{nr}{Z - \sum_{i=1}^{n} z_j}, \tag{11}$$

where $Z = \sum_{i=1}^{N} z_j$. In this example, there are no government expenditures, but all unemployed agents are entitled to unemployment benefits. The following two lemmas determine how the surplus of the n^{th} agent changes with n, the number of agents that are unemployed.

Lemma 2. *Assume that $r > 0$ and $z_j = \varepsilon z_{j-1}$ with $\varepsilon \geq 1$ for $j > 1$. Then $(\tau_{n+1} - \tau_n)$ is strictly increasing in n.*

Proof. Let $\Lambda_n = (1 + \varepsilon + \varepsilon^2 + \cdots + \varepsilon^n)$. Then the definition of the tax rate and simple algebra gives

$$\begin{aligned}
\tau_{n+1} - \tau_n &= r\left(\frac{n+1}{Z - \sum_{i=1}^{n+1} z_i} - \frac{n}{Z - \sum_{i=1}^{n} z_i}\right) \\
&= r\left(\frac{n+1}{Z - z_1(1 + \varepsilon + \varepsilon^2 + \cdots + \varepsilon^{n+1})} - \frac{n}{Z - z_1(1 + \varepsilon + \varepsilon^2 + \cdots + \varepsilon^{n})}\right) \\
&= r\left(\frac{n+1}{Z - z_1(\Lambda_n + \varepsilon^{n+1})} - \frac{n}{Z - z_1\Lambda_n}\right) \\
&= r\left(\frac{1}{Z - z_1(\Lambda_n + \varepsilon^{n+1})} + \frac{z_1 n \varepsilon^{n+1}}{\left(Z - z_1(\Lambda_n + \varepsilon^{n+1})\right)\left(Z - z_1\Lambda_n\right)}\right),
\end{aligned}$$

which is increasing in n. ■

Lemma 3. *Assume that $r > 0$ and $z_j = \varepsilon z_{j-1}$ with $\varepsilon \geq 1$ for $j > 1$. Let $\Gamma_{n+1} = (\varepsilon - 1)(1 - \tau_{n+1}) - (\tau_{n+1} - \tau_n)$. Then*

$$s_{n+1}(z_{n+1}) - s_n(z_n) = \Gamma_{n+1} z_n$$

and Γ_n is strictly decreasing in n.

Proof. Simple algebra gives that

$$\begin{aligned}
s_{n+1}(z_{n+1}) - s_n(z_n) &= \left[(\varepsilon - 1)(1 - \tau_{n+1}) - (\tau_{n+1} - \tau_n)\right] z_n \\
&= \Gamma_{n+1} z_n.
\end{aligned}$$

Lemma 2 shows that $(\tau_{n+1} - \tau_n)$ is increasing in n, which together with the property that τ_{n+1} is increasing in n proves that Γ_{n+1} is decreasing in n. ■

The lemmas show that the increase in tax rates *accelerates* as n increases and that this is reflected in the change in the surplus value of the marginal worker. The intuition for these results is that the linear increase in productivity levels is not enough to offset the accelerating effect of the reduction in the tax base.

The main result of this section is given in the following proposition.

Proposition 3. *Suppose that the following holds:*

- $\exists \overline{n}$ *such that* $s_{\overline{n}}(z_{\overline{n}}) = 0$ *and* $s_{\overline{n}+1}(z_{\overline{n}+1}) < 0$,
- *Assume that* $r > 0$ *and* $z_j = \varepsilon\ z_{j-1}$ *with* $\varepsilon \geq 1$ *for* $j \geq \overline{n} + 1$,

Then $s_k(z_k) < 0$ *for* $k > \overline{n} + 1$.

Proof. The first condition implies that $\Gamma_{\overline{n}+1} < 0$. Lemma 3 then implies that $\Gamma_{\overline{n}+2} < \Gamma_{\overline{n}+1} < 0$. This implies that

$$s_{\overline{n}+2}(z_{\overline{n}+2}) = s_{\overline{n}+1}(z_{\overline{n}+1}) + \Gamma_{\overline{n}+2}\ z_{n+1} < 0.$$

Simple iteration completes the proof. ■

The first assumption says it is an equilibrium for $\overline{n}$ workers to be unemployed and that workers with skill level $\overline{n} + 1$ are marginal workers. That is, if they become unemployed then the rise in the tax rate makes their surplus negative.[20]

The second assumption is a condition on how clustered the productivity levels are. One can replace this condition by weaker conditions such as $z_{j+1}/z_j \leq z_{\overline{n}+1}/z_{\overline{n}}$ for $j > \overline{n} + 1$. Key is that there are no "gaps" between the skill levels. The intuition behind this result is that if the increase in the tax rate caused by the $(n+1)^{th}$ type of worker becoming unemployed is sufficiently large to make employment unattractive for these workers, then the increase in the tax rate caused by the $(n+j)^{th}$ type of worker $(j > 1)$ is sufficient to make employment unattractive for them as well.

Note that the proposition does not say that the economy necessarily unravels, it only says that it could. In fact, the case with $n = \overline{n}$ $(< N)$ is by construction an equilibrium. The implication of the proposition is that if the distribution is sufficiently dense, then the economy could unravel.

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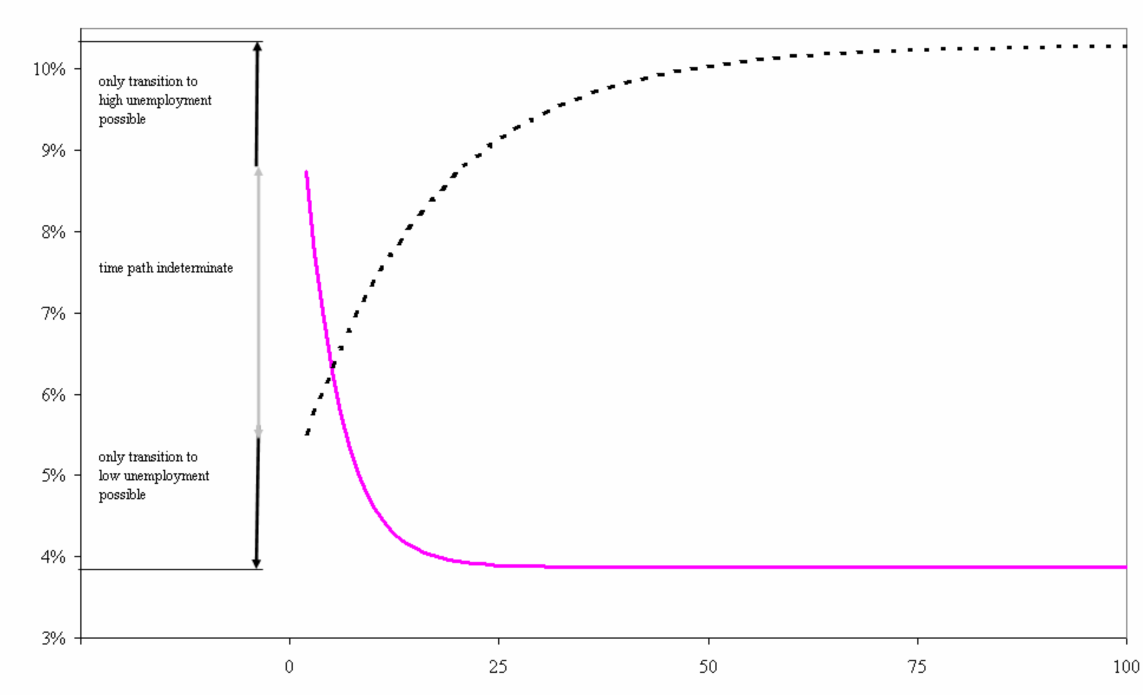
Figure 1*: Unemployment rates after a one-time burst in job destructions*



Notes: The parameters for this figure are given in Table 1.

Table 1: Parameter Values

g	0.35	λ_l	0.15
ψ	0.275	λ_h	0.5
r	0.444	ρ_l^x	0.05
b	0.257	ρ_h^x	0.0
$\phi_h = 1-\phi_l$	0.91	ρ^r	0.01
z_l	1.0	β	0.94
z_h	1.2		

Table 2: Steady-state properties

	low-unemployment steady state	**high-unemployment steady state**
unemployment rate	3.9%	10.3%
total govt. outlays/output	32.2%	36.8%
total net transfers/output	1.5%	4.3%