Accuracy of models with heterogeneous agents

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Introduction

Models with heterogeneous agents have many different dimensions
Krusell-Smith algorithm

- Numerical integration
  - Typically not a source of inaccuracy
    if you use quadrature & don't have too many sources of uncertainty
  - **Test**: Use more quadrature nodes and see whether results change
- Accuracy of individual policy rule (given aggregate rule)
  - **Test**: Use Euler equation errors; either at constructed fine grid or a points in simulation
- Accuracy of aggregate policy rule
  - **Test**: See below
- Be aware that errors of the the three blocks can interact with each other

Accuracy of aggregate law of motion

- Important to check its accuracy *without imposing functional form assumption*
- Given the complexity one typically has no choice but to use simulations to evaluate complete model
- So what are the inputs?
  - individual policy rule (fixed)
  - initial cross-sectional distribution over capital and employment status
  - a procedure to simulate the economy
  - a candidate aggregate law of motion which needs to be checked for accuracy

$$m_{t+1} = \bar{\phi}(z_{t+1}, z_t, m_t; \widehat{\alpha}) + u_{t+1} \tag{1}$$

$u_{t+1} \equiv 0$ for true transition law

Popular accuracy procedure

- Simulate a time series $\{m_t\}_{t=1}^{T}$ using *only* individual policy rules
- Use those values for m_t in LHS and RHS of (1) and check error
    - KS simulataneously estimates $\bar{\alpha}$ but this is not necessary
- Accuracy measure is the R^2 (and the standard error of the regression)

Problems

1. Overfitting: adding higher-order terms can only improve your accuracy measures
   - should not be big problem with sample is large enough
2. Bad to use same draw to estimate $\bar{\phi}(\cdot)$ and evaluate accuracy
   - should not be big problem with sample is large enough

Problems

3. R^2 and $\widehat{\sigma}_u$ are averages, which are weak measures
4. R^2 scales errors. That is the R^2 of (1) is substantially higher than the R^2 of the following *identical* regression equation:

$$\Delta m_{t+1} = m_{t+1} - m_t = \bar{\phi}(z_{t+1}, z_t, m_t; \widehat{\bar{\alpha}}) - m_t + u_{t+1}$$

5. Not clear when an R^2 is low

- inaccurate solution can easily have an R^2 above 0.999 (see below)

But the real problem is:

- "true" law of motion is used to generate explanatory variables
- that is, each period the truth is used to update the approximation

What does a modification to KS law of motion do to the R^2?

$$\ln K_t = \alpha_1 + \alpha_2 a_t + \alpha_3 \ln K_{t-1} + u_t$$

$$\alpha_3 = 0.96404$$

$$R^2 = 0.99999729$$

Experiment:

- Change α_3
- Adjust α_1 to keep mean of u_t equal to zero
- Recalculate R^2
- Calculate implied standard deviation of $\ln K_t$ to evaluate magnitude of the change

	R^2	implied standard dev
$\alpha_3 = 0.9604$ (original regression)	0.99999729	0.0248
$\alpha_3 = 0.954187$	0.99990000	0.0217
$\alpha_3 = 0.9324788$	0.99900000	0.0174
$\alpha_3 = 0.8640985$	0.99000000	0.0113

These updates in α_3 change the dgp considerably but not the R^2

Better accuracy procedure

1. Generate series independently
   using only same aggregate shocks and initial distribution
   1. As above generate a time series $\{m_t\}$ by simulating
   2. *Without using this time series* generate a new series using the candidate aggregate law of motion
2. Calculate the max and check at what kind of observation it occurs
3. Plot both time series = essential accuracy plot
4. Compare some properties of the two laws of motion
   e.g., impulse response functions

What did Krusell & Smith use?

- They emphasized the R^2 and $\hat{\sigma}_u$
- But they also looked at many other measues
  - alternative functional forms
  - economic arguments
  - *100 period-ahead forecast errors* (This turns out to be just as powerful as max of procedure proposed above)

Monte Carle example #1

Truth is given by

$$m_{t+1} = \alpha_0 + \alpha_1 m_t + \alpha_2 a_t + \alpha_3 m_{t-1}$$

Approximating law of motion

$$m_{t+1} = \bar{\alpha}_0 + \bar{\alpha}_1 m_t + \bar{\alpha}_2 a_t$$

Monte Carlo example #2

Truth is given by

$$\begin{aligned} m_{t+1} &= \alpha_0 + \alpha_{1,t} m_t + \alpha_2 a_t. \\ \alpha_{1,t} &= \left(\alpha_1 + \frac{\alpha_3}{\alpha_4 \exp(-\alpha_5 m_t)} \right) \end{aligned}$$

Approximating law of motion

$$m_{t+1} = \bar{\alpha}_0 + \bar{\alpha}_1 m_t + \bar{\alpha}_2 a_t$$

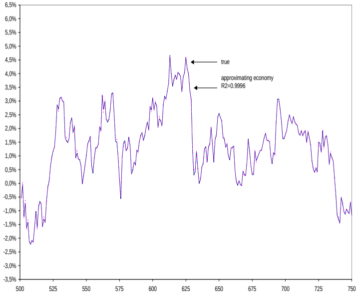
Traditional accuracy test

	#1.1	#1.2	#1.3	#1.4
R^2	0.9995	0.9940	0.99983	0.99981

minimum across Monte Carlo replications (that is, there are even higher ones)

Figure 1: In-sample fit of approximating law of motion

Panel A: Experiment 1.1



Panel B: Experiment 1.2

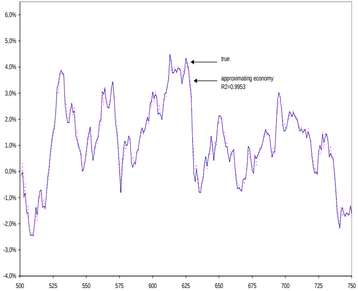
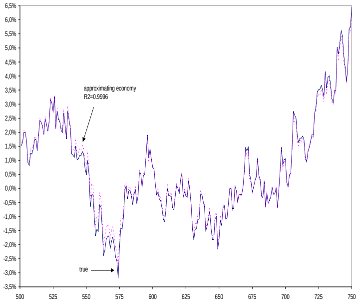


Figure 2: The essential accuracy plot – Separately generated series

Panel A: Experiment 1.1



Panel B: Experiment 1.2

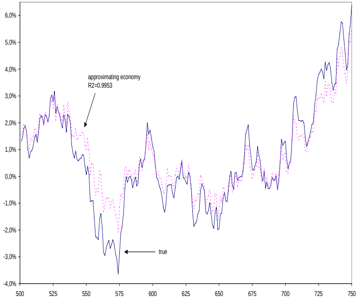
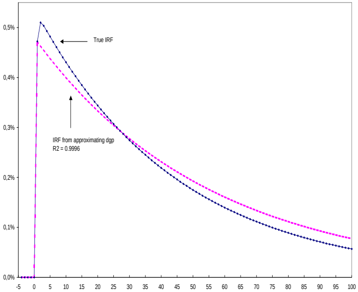


Figure 3: Impulse response functions

Panel A: Experiment 1.1



Panel B: Experiment 1.2

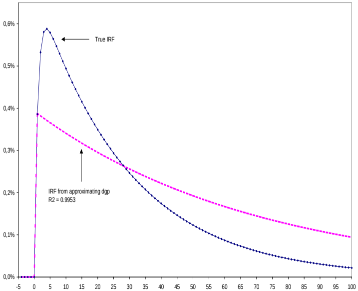
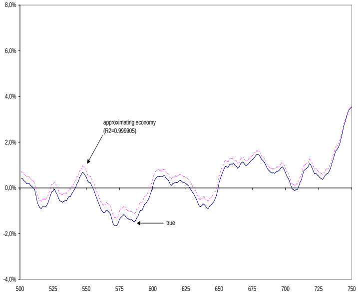


Figure 4: The essential accuracy plot – Separately generated series

Panel A: Experiment 2.1



Panel B: Experiment 2.2

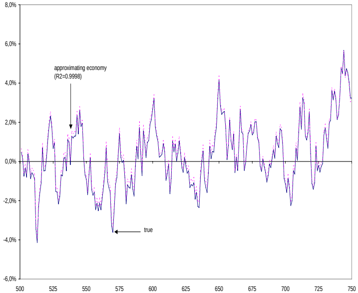


Figure 5: The essential accuracy plot – Separately generated series
Experiment 2.2 – part of simulation where maximum error occurs

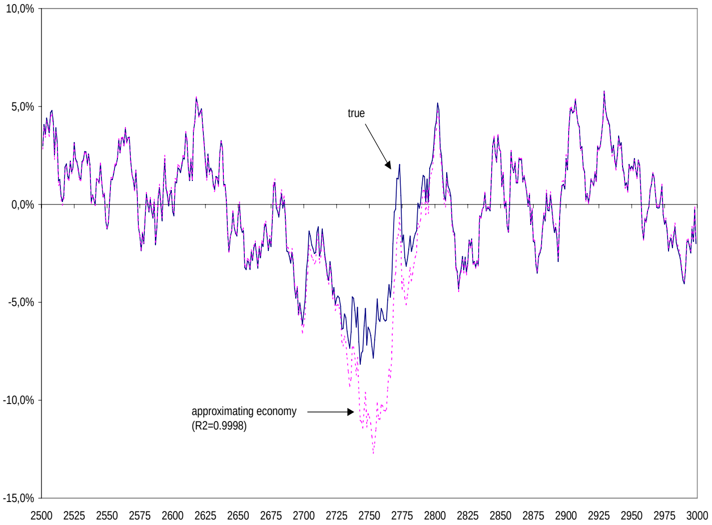
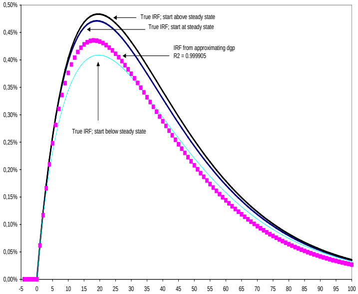


Figure 6: Impulse response functions

Panel A: Experiment 2.1



Panel B: Experiment 2.2

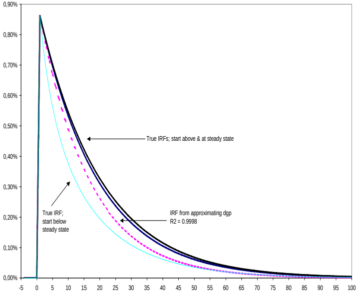


Figure 7: The essential accuracy plot – Separately generated series
Krusell-Smith economy

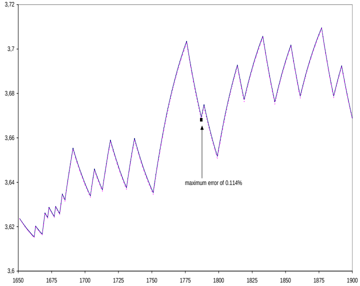
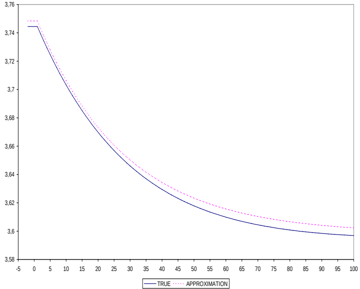


Figure 8: Impulse response function in the KS economy



New accuracy test

	#1.1	#1.2	#1.3	#1.4
$\widehat{u}^{\text{max}}$	0.83%	3.34%	1.86%	1.83%
$\widehat{u}^{\text{ave}}$	0.16%	0.67%	0.11%	0.12%

minimum across Monte Carlo replications, that is, even in the best Monte Carlo are the errors not that small

References

- Den Haan, W.J., 2010, Assessing the Accuracy of the Aggregate Law of Motion in Models with Heterogeneous Agents, Journal of Economic Dynamics and Control.