Misspecification in Macroeconomics: Difficult but Not Impossible to deal with

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Abstract

In this hopefully intuitive note, we explain why it is difficult to deal with misspecification and incompleteness of dynamic macroeconomic models in applied research. While standard econometric approaches are not suitable to do this, we sketch methods that can deal with misspecification of macroeconomic models, including the approach proposed in Den Haan and Drechsel (2018) that consists of adding Agnostic Structural Disturbances (ASDs) to model equations. That paper also documents that minor misspecification can have massive distortions for estimation outcomes making clear the damaging consequences of ignoring misspecification. This note accompanies a non-technical column in which we summarize the contributions of Den Haan and Drechsel (2018).

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1 Dynamic macroeconomic models

A macroeconomic model can be represented in two different ways. The first is the set of equations characterizing the behavior and interaction of the different agents. A general formulation is given by

$$\mathbb{E}_t \left[f \left(y_{t+1}, y_t, y_{t-1}, \varepsilon_{t+1}, \varepsilon_t; \psi \right) \right] = 0. \tag{1}$$

$\mathbb{E}_t [\cdot]$ is an expectations operator and y_t is a vector that contains the endogenous variables of the model, such as aggregate output, consumption, and employment. Since we focus on business cycles, we define all variables as deviations from a trend. Thus, the unconditional means of all variables are equal to zero. When lagged values, y_{t-1}, appear in the model equations, then the past matters for the outcome in the current period, i.e., y_t. The presence of the future variables y_{t+1} and ε_{t+1} captures the forward looking behavior present in many macroeconomic models.[1]

ε_t is a vector containing the exogenous random variables of the model. These are the reason for business cycle fluctuations in most modern business cycle models. Since they are part of the model equations, they are called *structural* disturbances. By contrast, measurement error is part of an empirical specification, but is not part of the theoretical model. Popular structural disturbances are productivity disturbances, preference disturbances, and disturbances to monetary and fiscal policy. The law of motion for ε_t is given by

$$\varepsilon_{t+1} = \Gamma \varepsilon_t + e_{t+1}, \tag{2}$$

where e_t is a vector of structural innovations. These have zero mean and variance-covariance matrix Ω. Typically, it is assumed that Γ and Ω are diagonal matrices, which implies that the structural disturbances are not correlated with each other.

In a consistent system, the dimension of $f(\cdot; \psi)$, that is, the number of equations, is equal to the number of endogenous variables, that is, the dimension of y_t.

When estimating a model, researchers' have to make choices for model elements such as utility functions, financial frictions, etc.. These imply a functional form for $f(\cdot; \psi)$. The model's parameters, ψ, control key economic properties such as the degree of risk aversion, the degree of price indexation of wage contracts, and the substitutability of different produced goods.

The second way to represent a macroeconomic model is to use the solution to the system of equations given in equation (1). Such a solution expresses this period's outcome, y_t, as a function of the predetermined variables, y_{t-1} and the exogenous variables, ε_t. Thus,

$$y_t = g \left(y_{t-1}, \varepsilon_t; \psi \right). \tag{3}$$

Typically, one has to use numerical approximation methods to obtain an accurate representation for this function.

[1] Leads and lags could exceed one period, but there is no need to consider that possibility for the current analysis.

2 What could be wrong?

We can distinguish two types of misspecification. The first is that the model itself is incorrect. That means that the functional form $f(\cdot;\psi)$ or its implied law of motion $y_t = g(\cdot;\psi)$ is not a correct representation for observed data. The second is that the model itself is correct, but is incomplete. Models are simplifications of reality and, thus, are unlikely to capture all possible aspects that are relevant for the variables being studies. We will consider both problems.

Misspecification due to wrong functional forms. The econometrics literature has studied the properties of estimators when an empirical model such as $g(\cdot;\psi)$ is misspecified.[2] However, this literature has limited use for applied macroeconomics. The reason is that the objective of applied macroeconomics is typically not to get a good timeseries representation for y_t or an estimator that ensures convergence to a well-defined limit according to some statistical loss function. Estimated macroeconomic models are used to understand the underlying model and to perform policy experiments. Thus, it is important to uncover the true functional form and the true parameter values.

A more useful approach to detect misspecification is to consider a set of several theoretical models and to compare their empirical performance. But even if one considers a large set, then one only considers a fraction of all possible models. As discussed below, one could also compare a theoretical model with a reduced-form specification. The flexibility of reduced-form empirical models gives them an empirical advantage. Thus, a proper comparison would require a model selection criterion that imposes a suitable penalty on the reduced-form model. However, if the procedure prefers the reduced-form model (or a combination of the theoretical model and the reduced-form model), then one still doesn't have a model with which one easily can do policy analysis, since one doesn't know how the estimated reduced-form model is affected by policy experiments.[3]

Misspecification due to missing elements. Obviously, macroeconomic models are not unique in that they do not capture *all* relevant mechanisms that cause movements in the variables of interest. Each year, this reality is pointed out to thousands of undergraduate students during their first econometrics course. The solution of the econometrician is to add a regression error term. Doing so would change equation (3) to

$$y_t = g(y_{t-1}, \varepsilon_t; \psi) + u_t. \tag{4}$$

[2] An important early contribution is White (1982).

[3] According to the Lucas critique, one cannot expect reduced-form empirical relationships between variables observed during a period with particular government policies in place to remain unchanged if these policies (or in general aspects of the environment) do not remain the same.

This simple way out does not work for applied macroeconomics! There are several reasons. First, $f(\cdot;\psi)$ and, thus, $g(\cdot;\psi)$ represent the behavior of agents in a particular environment. It captures the economy's responses to changes in the elements of ε_t. To understand how agents respond to a particular disturbance, you have to understand the *complete* environment agents face.

As a thought experiment, consider an economic agent who faces two types of risk, say employment risk and house price risk. Suppose that an economic model $y_t = \widetilde{g}(y_{t-1}, \varepsilon_{1,t}; \psi)$ correctly models this agent's behavior if she faces employment risk, $\varepsilon_{1,t}$, but no house price risk. Would it be correct to use a regression residual to capture the missing element, that is, the role of house price risk? In general, the answer is no. There are two reasons. First, how agents' choices respond to changes in their employment status could very well depend on whether they also face house price risk. Specifically, economic agents who become unemployed are likely to cut into their savings by less (and thus reduce consumption by more) when there is a chance the value of their house might drop. More formally, the relationship between y_t and employment status, as captured by the function $y_t = \widetilde{g}(y_{t-1}, \varepsilon_{1,t}; \psi)$, will typically depend on whether there are other risk factors in the economy.

There is another reason why standard regression residuals are not a suitable solution to capture the role of missing disturbances in macroeconomic models. The reason is that they are unlikely to have the necessary property for the estimator to be consistent, namely being uncorrelated with the explanatory variables. The cause for this is related to the fact that macroeconomic models are dynamic.

Again, suppose that the true model is such that fluctuations in y_t are due to two disturbances, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ and that we can express y_t as follows:

$$y_t = y_{1,t} + y_{2,t}, \tag{5}$$

where $y_{1,t}$ captures the fluctuation due to $\varepsilon_{1,t}$ and $y_{2,t}$ the fluctuations due to $\varepsilon_{2,t}$. To give the approach to capture missing elements with a regression residual the best possible chance we assume that $y_{1,y}$ and $y_{2,t}$ are completely independent of each other. To make clear that this second problem is not related to nonlinearities, we assume that the model is linear. This means we have the following system of equations to characterize $y_t = g(y_{t-1}, \varepsilon_t; \psi)$:

$$\begin{aligned} y_t &= y_{1,t} + y_{2,t}, \\ y_{1,t} &= A_1(\psi)\, y_{1,t-1} + B_1(\psi)\, \varepsilon_{1,t}, \\ y_{2,t} &= A_2(\psi)\, y_{2,t-1} + B_2(\psi)\, \varepsilon_{2,t}, \end{aligned} \tag{6}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are independent.

The problem is that we only observe y_t, not its two components $y_{1,t}$ and $y_{2,t}$. So what would happen if a macroeconomist would attempt to estimate and/or test a model for y_t, when this model only includes the model for $y_{1,t}$ and a regression residual to capture the missing part. That empirical specification would be equal to

$$y_t = A_1(\psi)\, y_{t-1} + B_1(\psi)\, \varepsilon_{1,t} + u_t, \tag{7}$$

where y_{t-1} and $\varepsilon_{1,t}$ are the explanatory variables and u_t the regression residual. For simplicity, we assume that the econometrician observes both y_t and $\varepsilon_{1,t}$ without measurement error. Using the equations of the true underlying model, we get the following system:

$$\begin{aligned} y_t &= y_{1,t} + y_{2,t} \\ &= A_1(\psi)\, y_{1,t-1} + B_1(\psi)\, \varepsilon_{1,t} + A_2(\psi)\, y_{2,t-1} + B_2(\psi)\, \varepsilon_{2,t} \\ &= A_1(\psi)\,(y_{1,t-1} + y_{2,t-1}) + B_1(\psi)\, \varepsilon_{1,t} \\ &\quad +(A_2(\psi) - A_1(\psi)) y_{2,t-1} + B_2(\psi)\, \varepsilon_{2,t} \\ &= A_1(\psi)\, y_{t-1} + B_1(\psi)\, \varepsilon_{1,t} + u_t \\ &\text{with} \\ u_t &= (A_2(\psi) - A_1(\psi)) y_{2,t-1} + B_2(\psi)\, \varepsilon_{2,t}. \end{aligned} \tag{8}$$

Using the equations of the true underlying model, we have derived an equation that is identical to the empirical specification given in equation (7). Nevertheless, there is a problem. The problem is that the regression residual u_t is correlated with the explanatory variable y_{t-1}. This means that standard regression analysis would lead to inconsistent estimates.[4] Note that this is true even if $\varepsilon_{2,t}$, i.e., the missing disturbance, is itself not serially correlated. For u_t to be uncorrelated with the explanatory variables one would need that A_1 and A_2 are identical to each other, $\varepsilon_{2,t}$ is not serially correlated, and the two mechanisms are independent from each other. One can only expect such strong assumptions to be valid if the missing element $\varepsilon_{2,t}$ represents pure data measurement error. But it is unlikely that the only reason for the gap between observed data and economic models is the presence of measurement errors.

3 Dealing with misspecificaiton

Conceptually, it is actually not that difficult to deal with misspecification. Key in doing it right is the understanding that theoretical models explain at best a subset of the components driving y_t and that these components are not observed.

Comprehensive misspecification procedure. Let's return to the case where an applied macroeconomist has a model that is based only on one disturbance $\varepsilon_{1,t}$.[5] Suppose it is understood that there are other structural disturbances driving the data. A

[4] In theory, instrument variables would work. However, it is difficult to find exogenous observables suitable to serve as instruments given that macroeconomic variables are jointly determined.

[5] Nothing depends on $\varepsilon_{1,t}$ having only one element, but it possibly helps in understanding the argument.

sensible empirical specification dealing with missing elements would be the following

$$\begin{aligned} y_t &= y_{1,t} + y_{2,t}, \\ y_{1,t} &= A_1(\psi)\, y_{1,t-1} + B_1(\psi)\, \varepsilon_{1,t}, \\ y_{2,t} &= \boldsymbol{A}_2 y_{2,t-1} + \boldsymbol{B}\boldsymbol{\varepsilon}_{2,t}, \end{aligned} \tag{9}$$

where the bold symbols indicate that these are reduced-form objects not based on theory. The existing approaches in the literature dealing with misspecification in macroeconomic models are based on this setup.[6] Moreover, this procedure does not only deal with misspecification due to missing elements. It can also deal with the misspecification of the model itself, since the reduced form can completely take over all explanatory power if the estimation results in a zero standard deviation for the productivity disturbance $\varepsilon_{1,t}$.

Why is $y_{2,t}$ different from a residual? The reason is that $y_{2,t}$ does not affect the explanatory variable, here $y_{1,t}$, whereas a regression error term does, as is made clear in equation (8).

What is done in the literature? Although there is a sensible approach to deal with misspecification, it is rarely used. Admittedly, the approach outlined above has some disadvantages. The objective of applied macroeconomics is typically not to come up with the best time-series representation of y_t, but to come up with a theoretical structure to *understand* the real world. Given the flexibility of the reduced-form component, it may limit the role for the theoretical part of the empirical specification. As pointed out above, the reduced-form part of the model is not immune to the Lucas critique so the estimated model would have limited value to study the effect of changes in the model environment such as changes in government policy. Moreover, the reduced-form block could be a bad representation of reality if the set of available observables does not include all elements of y_t, that is, all variables in the true economy needed to model the dynamics.[7] Another drawback of the available misspecification approach is that it does complicate the empirical analysis.

Fortunately, applied macroeconomists understand that one cannot simply add a regression residual. But if one also does not want to incorporate a reduced-form parallel block, then one has to include enough disturbances to ensure that one has a complete characterization of all forces driving the variables of interest. This is indeed the popular path followed. It often means having a very long list of structural disturbances and no serious consideration of misspecification.

To us, it seems impossible to get all structural disturbances right and this belief motivated us to investigate the consequences of including the wrong structural distur-

[6] See Ireland (2004), Del Negro, Schorfheide, Smets, and Wouters (2007), and Del Negro and Schorfheide (2009).

[7] By contrast, the structure imposed by the theory makes it easier to deal with unobserved elements in y_t.

bances in the empirical specification and to search for a procedure that can detect and correct for the misspecification of structural disturbances. In Den Haan and Drechsel (2018), we document that even minor misspecifications can have devastating consequences. We also develop an alternative, Agnostic Structural Disturbances (ASDs) to detect and correct for misspecification of structural disturbances.

Agnostic Structural Disturbances (ASDs). Adding, for example, two ASDs would mean adding a 2×1 exogenous disturbance to *each* of the model equations. This means that the economic model as represented in equation (1) changes to

$$\mathbb{E}_t \left[f \left(y_{t+1}, y_t, y_{t-1}, \varepsilon_{1,t+1}, \varepsilon_{1,t}; \psi \right) \right] + \widehat{\boldsymbol{\Upsilon}} \varepsilon_{2,t} = 0, \tag{10}$$

where $\varepsilon_{1,t}$ contains the regular structural disturbances and $\varepsilon_{2,t}$ the ASDs. The matrix $\widehat{\boldsymbol{\Upsilon}}$ captures the impact of the ASDs on *all* model equations. Dynare, the software package typically used to estimate dynamic macroeconomic models, uses this type of model specification, so could easily incorporate ASDs.

It might seem that $\varepsilon_{2,t}$ is just like a standard regression residual in that it is not part of the model. But that is not true, it really is a *structural* disturbance that propagates through the economy according to the mechanisms described by $f(\cdot; \psi)$. The easiest way to understand this is to linearize the system given in equation (10). Then it can be shown that the implied solution has the following form:

$$\begin{aligned} y_t &= y_{1,t} + y_{2,t}, \\ y_{1,t} &= A_1(\psi)\, y_{1,t-1} + B_1(\psi)\, \varepsilon_{1,t}, \\ y_{2,t} &= A_1(\psi)\, y_{2,t-1} + \boldsymbol{B_2} \varepsilon_{2,t}. \end{aligned} \tag{11}$$

The component of y_t driven by fluctuations in $\varepsilon_{2,t}$, i.e., $y_{2,t}$, has the same AR coefficient as the component driven by fluctuations in the regular structural disturbance, $\varepsilon_{1,t}$.[8] Note that this coefficient depends on the structural parameters of the model, ψ. That is, the impact of $\varepsilon_{2,t}$ propagates through the system according to the economic mechanisms of the model just as regular structural disturbances do. The only difference between a regular structural disturbance and an ASD is that the *initial* impact of $\varepsilon_{2,t}$ is not constrained by the economic model, whereas the impact of $\varepsilon_{1,t}$ is.

Interpreting ASDs. In contrast to regular structural disturbances, ASDs have an unrestricted initial impact and do not impose any restrictions on the model. Since no specific theoretical reasoning is used to enter them into the model, they can capture *any* structural disturbance. After the model with ASDs has been estimated, however, one can use the ASDs' impulse response functions and the estimates of the ASDs' associated coefficients to see whether one can give an economic interpretation to the

[8] In fact, all structural disturbances propagate through the system in the same way in a *linearized* framework.

included ASDs. Den Haan and Drechsel (2018) document that this can be done using the model and data set of Smets and Wouters (2007).

Comparison to wedges. Adding disturbances to model equations is not a novel idea. Specifically, this idea is introduced in Chari, Kehoe, and McGrattan (2007) and they refer to these disturbances as wedges. The difference between wedges and ASDs is that ASDs are added to *all* model equations, whereas wedges are introduced to one or a small subset of equations that are selected by the researcher.[9] The consequence is that wedges do impose theoretical (zero) restrictions and their interpretation is most likely related to the chosen equation(s) in which the wedge is allowed to enter. In fact, wedges are given different labels, such as the labor wedge, depending on the equations in which they are entered. ASDs can only be interpreted and given a label *after* estimation.

Comparison to the alternative approach with a full reduced-form block. Our ASD approach is similar in spirit to the approach given in equation (9) pioneered in Ireland (2004) and formally developed in Del Negro, Schorfheide, Smets, and Wouters (2007) and Del Negro and Schorfheide (2009). However, this approach has a parallel model block that is fully reduced form with many additional parameters to estimate. With ASDs, only the initial impact is described with reduced-form coefficients. How these shocks propagate through the system is driven by the economic mechanisms of the model. This has both a disadvantage and a disadvantage. The disadvantage is that ASDs are limited in the types of misspecification they can detect and correct for. The advantage is that the more concise specification makes the ASD approach less reduced-form, more efficient and very easy to implement.

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[9]Den Haan and Drechsel (2018) use statistical model selection criteria to obtain more concise specifications in which the included ASDs are excluded from some equations. But this procedure is purely driven by the data and not by any prior theoretical considerations.

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